Magic Squares and the Magic of TI Technology

Live Webinar for Teachers Teaching with Technologies
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What is a magic square?

The **sum** of the numbers in each **row**, **column** and **main diagonal** is the **same**. An n by n magic square is said to be **normal** if the numbers used to make the square are 1, 2, 3, ..., n^2.

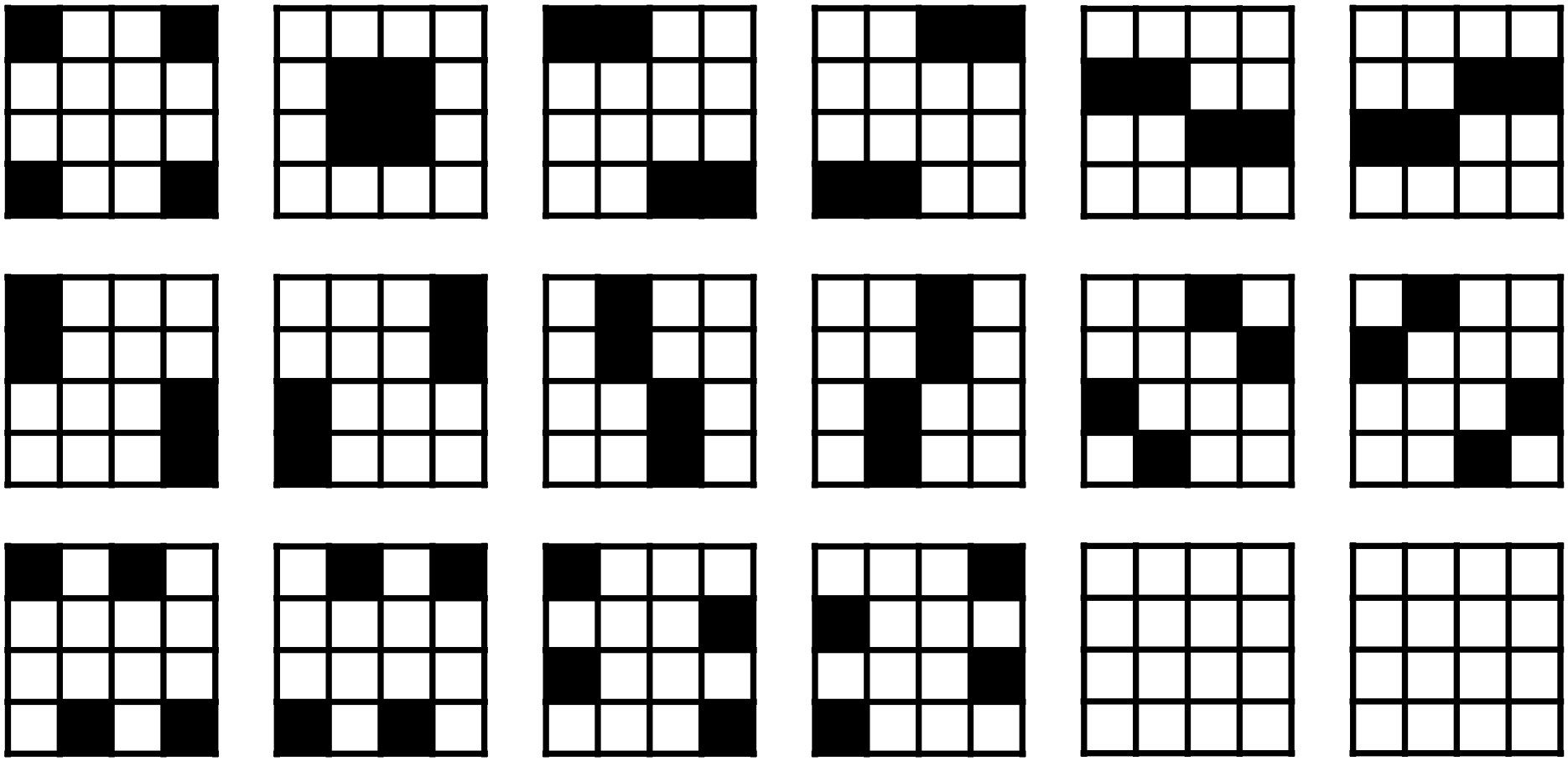
Shown below is a normal 4 x 4 magic square whose magic sum is 34.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

There are other combinations of 4 numbers that add up to 34. How many can you find?

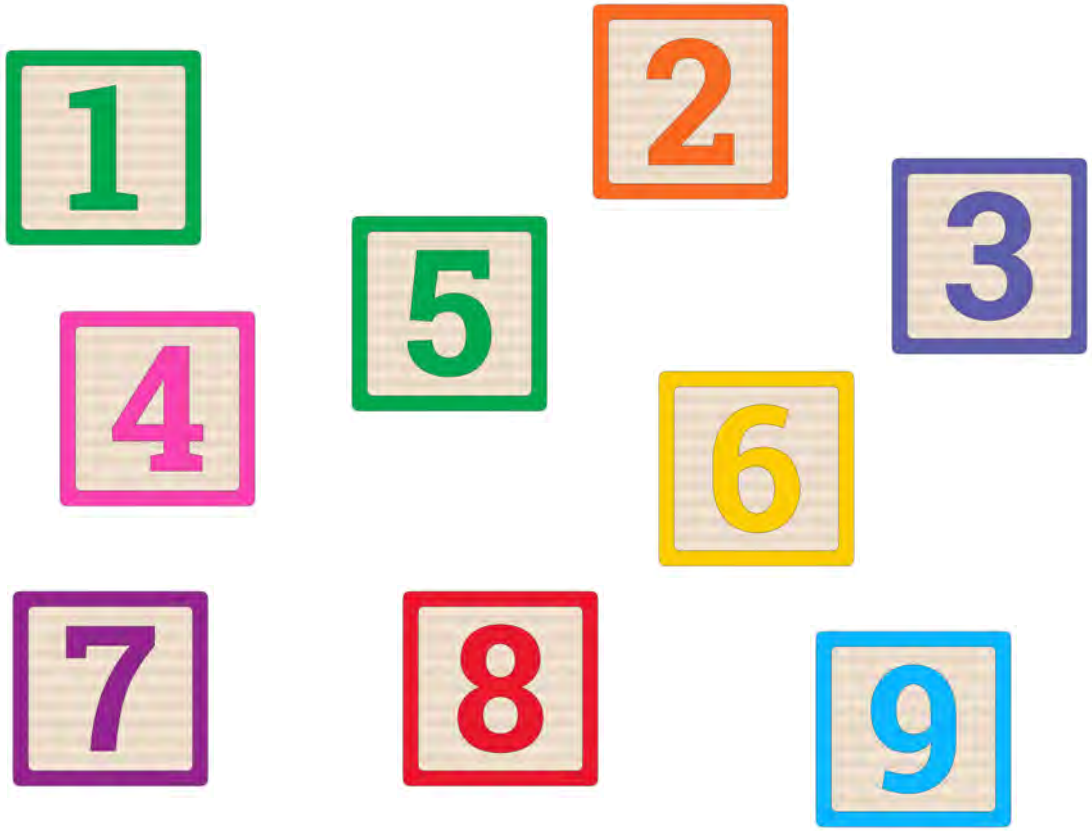
16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

A	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P



Are there any other combinations?

Make a 3 x 3 magic square using the numbers 1, 2, 3, ..., 9.



Make a 3 x 3 magic square using the numbers 1, 2, 3, ..., 9.

Let S be the magic sum.
The sum of the numbers in row 1 is S, the sum of the numbers in row 2 is S and the sum of the numbers in row 3 is S.

$3S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$
$3S = 45$
$S = 15$

This is amazing that we know the magic sum without making the square yet!

Let's find all the ways of making 15 using 3 of the numbers from 1 to 9.

8 + 3 + 4

Let's find all the ways of making 15 using 3 of the numbers from 1 to 9.

8	+	3	+	4
1	+	5	+	9
6	+	7	+	2
8	+	1	+	6
3	+	5	+	7
4	+	9	+	2
8	+	5	+	2
6	+	5	+	4

What do you notice?

Let's find all the ways of making 15 using 3 of the numbers from 1 to 9.

8	+	3	+	4
1	+	5	+	9
6	+	7	+	2
8	+	1	+	6
3	+	5	+	7
4	+	9	+	2
8	+	5	+	2
6	+	5	+	4

	5	

The number 5 is the only number that can be placed in the middle of the square.

Let's find all the ways of making 15 using 3 of the numbers from 1 to 9.

8	+	3	+	4
1	+	5	+	9
6	+	7	+	2
8	+	1	+	6
3	+	5	+	7
4	+	9	+	2
8	+	5	+	2
6	+	5	+	4

8	1	6
3	5	7
4	9	2

Are there are other 3 x 3 normal magic squares? How do you know?

Starting at the number **1**, **move your finger** to the number **2**, then to **3**, then to **4**, all the way to **9**. What patterns do you see in the movement of your finger?

Use your patterns to make a **5 x 5 magic square**.

8	1	6
3	5	7
4	9	2

8	1	6
3	5	7
4	9	2

8	1	6
3	5	7
4	9	2

		1		
	5			
4	6			
				3
			2	

8	1	6
3	5	7
4	9	2

		1	8	
	5	7		
4	6			
10				3
11			2	9

8	1	6
3	5	7
4	9	2

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Magic constant for a **5 x 5** normal magic square is **65**

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

USE MATH TO MAKE MAGIC SQUARES

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Coding project for your students!
Write a TI-Basic or TI-Python program that creates a magic square on an odd by odd chessboard.

Doubly-even (4 x 4, 8 x 8, 12 x 12) magic squares are more difficult to make than odd by odd magic squares. Singly-even (6 x 6, 10 x 10, 14 x 14) magic squares are very tough to make.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Magic constant is 34

6	32	3	34	35	1
7	11	27	28	8	30
19	14	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

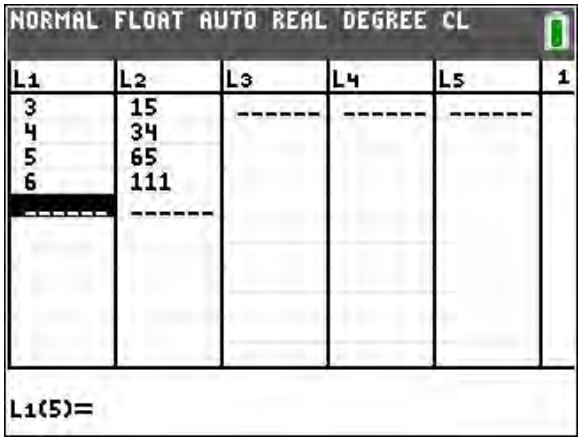
Magic constant is 111

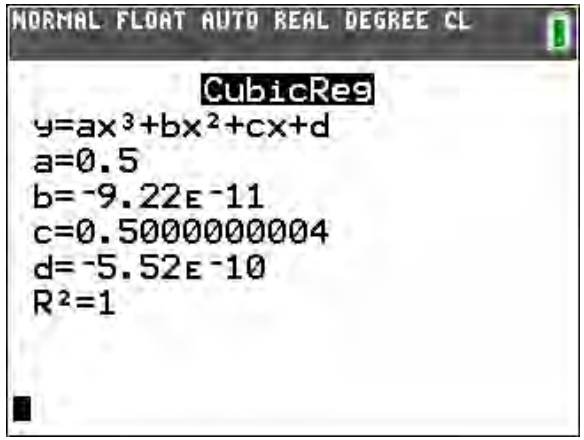
Determine the **magic constant** for an ***n* x *n*** normal magic square in two different ways.

n	magic constant S
3	15
4	34
5	65
6	211

Determine the **magic constant** for an ***n* x *n*** normal magic square in two different ways.

Method 1





$$S = 0.5n^3 + 0.5n$$

Determine the **magic constant** for an ***n* x *n*** normal magic square in two different ways.

Method 2 (same as before)

$$nS = 1 + 2 + 3 + \ \dots \ + \ n^2$$

$$nS = \frac{n^2(n^2 + 1)}{2}$$

$$S = \frac{n(n^2 + 1)}{2}$$

Let's think of a magic square as being a matrix. What are some **power**ful questions that we can ask about these matrices?

Let's think of a magic square as being a **matrix**. What are some **power**ful questions that we can ask about these matrices?

Is A^n a magic square where n is a positive integer?
Is A^{-1} a magic square?
Is the square root of a matrix a magic square?
Is e^A a magic square?

There are too many questions to answer in a one-hour webinar. So let's focus on **odd** powers of 3 x 3 magic squares/matrices.

1.2 | 1.3 | 1.4 | *Magic Sq (1) | RAD

$$\begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix} \rightarrow \mathbf{a}$$ is a magic square.

The magic sum is 15.

$$\mathbf{a}^3 \rightarrow \mathbf{b} = \begin{bmatrix} 1149 & 1029 & 1197 \\ 1173 & 1125 & 1077 \\ 1053 & 1221 & 1101 \end{bmatrix}$$ is also a magic square. The magic sums is 3,375 = 15^3.

1.3 | 1.4 | 1.5 | *Magic Sq (1) | RAD

$$\mathbf{a}^5 \rightarrow c = \begin{bmatrix} 252549 & 255429 & 251397 \\ 251973 & 253125 & 254277 \\ 254853 & 250821 & 253701 \end{bmatrix}$$ is a magic square. Magic sum is 759,375 = 15^5.

$$\mathbf{a}^7 = \begin{bmatrix} 56966949 & 56897829 & 56994597 \\ 56980773 & 56953125 & 56925477 \\ 56911653 & 57008421 & 56939301 \end{bmatrix}$$ is a magic square. Magic sum is 170,859,375 = 15^7.

1.8 1.9 1.10 Magic Sq (1) RAD

$$\begin{bmatrix} u-v & u+v+w & u-w \\ u+v-w & u & u-v+w \\ u+w & u-v-w & u+v \end{bmatrix} \rightarrow \mathbf{d} \text{ magic square}$$

The magic sum is 3u.

$\mathbf{d}^3$ = e is also a magic square.

The magic sum is $27u^3$.

Here are the elements of matrix e:

1.9 1.10 1.11 *Magic Sq (1) RAD

e11 = $9u^3 - 3v^3 + 3vw^2$

e12 = $9u^3 + 3(v^2 - w^2)(v + w)$

e13 = $9u^3 - 3v^2 w + 3w^2$

e21 = $9u^3 + 3(v^2 - w^2)(v - w)$

e22 = $9u^3$

e23 = $9u^3 + 3(v^2 - w^2)(v - w)$

1.10 1.11 1.12 *Magic Sq (1) RAD

e31 = $9u^3 + 3v^2 w - 3w^2$

e33 = $9u^3 - 3(v^2 - w^2)(v + w)$

e33 = $9u^3 + 3v^3 - 3vw^2$

1.15 1.16 1.17 *Magic Sq (1) RAD

$$f = \begin{bmatrix} s+t+p & p-2\cdot s & s-t+p \\ p-2\cdot t & p & 2\cdot t+p \\ -s+t+p & 2\cdot t+p & -s-t+p \end{bmatrix}$$

is a square matrix.

The magic sum is 3p.

$\mathbf{f}^3 = g$ is also a square matrix.

The magic sum of $\mathbf{f}^3 = g$ is $27p^3$.

Here are the elements of matrix g:

1.16 1.17 1.18 *Magic Sq (1) RAD

$g11 = 9p^3 + 12s^2t + 12st^2$

$g12 = 9p^3 - 24s^2t$

$g13 = 9p^3 + 12s^2t - 12st^2$

$g21 = 9p^3 - 24s^2t$

$g22 = 9p^3$

$g23 = 9p^3 + 24s^2t$

1.17 1.18 1.19 *Magic Sq (1) RAD

$g31 = 9p^3 - 12s^2t + 12st^2$

$g32 = 9p^3 + 24s^2t$

$g33 = 9p^3 - 12s^2t - 12st^2$

If the inverse of a magic square/matrix exists, is it also a magic square?

1.4 1.5 1.6 *Magic Sq (2) RAD

The determinant of A = det(A) = 360.

$$A^{-1} = (1/360)\begin{bmatrix} 23 & -52 & 53 \\ 38 & 8 & -22 \\ -37 & 68 & -7 \end{bmatrix} = (1/360)\text{Adj}(A)$$

Adj(A) is a magic square. Magic number = 24

A^{-1} is a magic square with magic number =

24 / 360 = 1 / 15 = 1 / magic number of A.

Creating a new magic square from a magic square

1.1 1.2 1.3 Magic Sq (2) RAD

Matrix A = $\begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$ is a magic square.

The magic sum is 15.

Create a new 3 x 3 matrix by replacing each element with the sum of all elements that are in a different row and column. For example, replace a11 = 8 with a22 + a23 + a32 + a33 = 5 + 7 + 9 + 2. Do this for each element in A.

1.1 1.2 1.3 *Magic Sq (2) RAD

The new matrix B created is a magic square:

B = $\begin{bmatrix} 23 & 16 & 21 \\ 18 & 20 & 22 \\ 19 & 24 & 17 \end{bmatrix}$ and the magic sum is 60.

C = $\begin{bmatrix} u-v & u+v+w & u-w \\ u+v-w & u & u-v+w \\ u+w & u-v-w & u+v \end{bmatrix}$ is a magic square. The magic sum is 3u. Apply this transformation to matrix C to create matrix D:

1.1 1.2 1.3 *Magic Sq (2) RAD

D = $\begin{bmatrix} 4u-v & 4u+v+w & 4u-w \\ 4u+v-w & 4u & 4u-v+w \\ 4u+w & 4u-v-w & 4u+v \end{bmatrix}$ is a magic square with a magic sum of 12u.

Matrix A has u = 5, v = −3, and w = −1.

Replacing those values in matrix C will generate matrix B. They also confirm the relationship between the magic sums.

1.2 1.3 1.4 *Magic Sq (2) RAD

If we apply this procedure again to matrix B we obtain matrix E = $\begin{bmatrix} 83 & 76 & 81 \\ 78 & 80 & 82 \\ 79 & 84 & 77 \end{bmatrix}$

which is a magic square with magic sum = 240

The magic sum of matrix E is four times the magic sum of matrix E, as expected.

One of **Benjamin Franklin's** magic squares

1.1 1.2 1.3 Magic Sq (3) RAD

$$\begin{bmatrix} 52 & 61 & 4 & 13 & 20 & 29 & 36 & 45 \\ 14 & 3 & 62 & 51 & 46 & 35 & 30 & 19 \\ 53 & 60 & 5 & 12 & 21 & 28 & 37 & 44 \\ 11 & 6 & 59 & 54 & 43 & 38 & 27 & 22 \\ 55 & 58 & 7 & 10 & 23 & 26 & 39 & 42 \\ 9 & 8 & 57 & 56 & 41 & 40 & 25 & 24 \\ 50 & 63 & 2 & 15 & 18 & 31 & 34 & 47 \\ 16 & 1 & 64 & 49 & 48 & 33 & 32 & 17 \end{bmatrix} \rightarrow B$$

B is a square created by Ben Franklin.

1.1 1.2 1.3 Magic Sq (3) RAD

Every row and column sum is 260.

Every bent diagonal sum is 260.

The four corner numbers plus the four middle numbers sum to 260.

Examples of a bent diagonal sum are:

b14 + b23 + b32 +b41+b51+ b62 + b73 + b84

b21 + b12 + b38 +b47+b57+ b68 + b71 + b82

1.1 1.2 1.3 Magic Sq (3) RAD

Any 2 x 2 sub square sums to 130.

The sum of any four numbers equidistant from the center of the square is 130.

Examples of four such numbers are:

b41 + b51 + b48 + b58

b22 + b72 + b27 + b77

b32 + b62 + b37 + b67

Two more powerful and radical questions.

Is it possible to calculate ***e*** to the **power of a matrix**? Is it possible to calculate the **square root of a matrix**? If so, are they magic squares?

$$e^{\begin{pmatrix} 8 & 1 & 6 \\ 4 & 9 & 2 \\ 3 & 5 & 7 \end{pmatrix}} \qquad \sqrt{\begin{pmatrix} 8 & 1 & 6 \\ 4 & 9 & 2 \\ 3 & 5 & 7 \end{pmatrix}}$$

A semi-magic knight's tour on an 8 x 8 chessboard

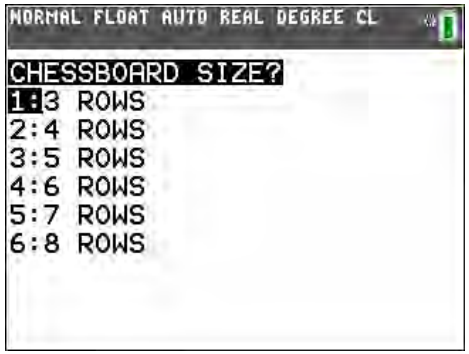
38	35	62	25	60	23	10	7
63	26	37	34	11	8	59	22
36	39	28	61	24	57	6	9
27	**64**	33	40	5	12	21	58
50	29	4	13	48	41	56	19
1	14	49	32	53	20	47	44
30	51	16	3	42	45	18	55
15	2	31	52	17	54	43	46

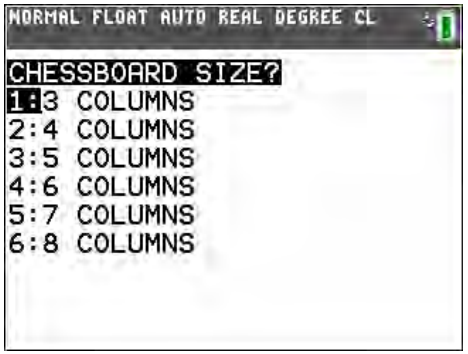
For over 150 years no one was able to find a magic knight's tour and no one knew if it was possible.

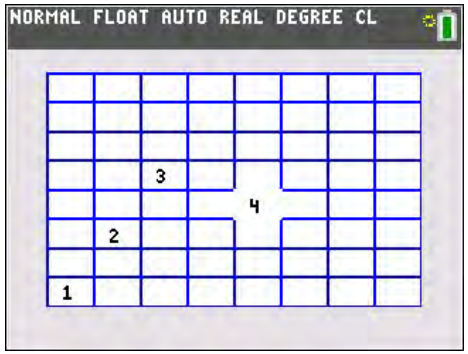
The issue was settled on August 6, 2003 after an international team determined that the answer is

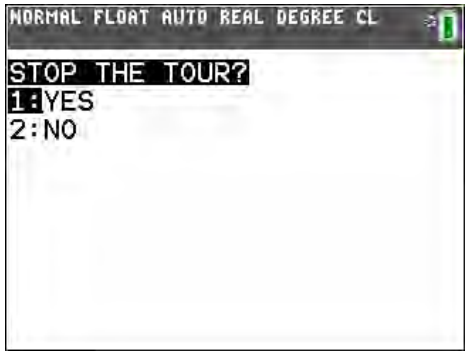
NO

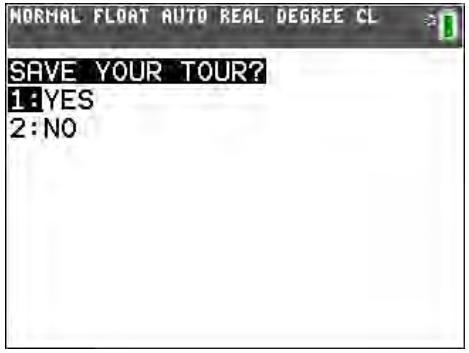
Coding project for your students!
Write a TI-Basic or TI-Python program that can be used by an individual to record, save and edit attempts to make a knight's tour on chessboards of different sizes.

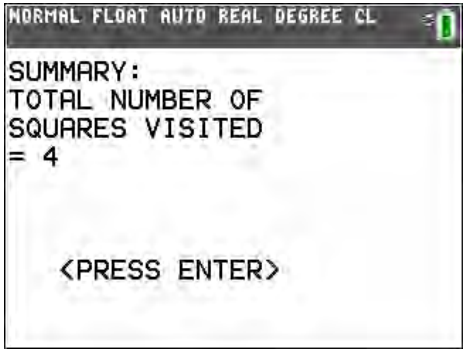


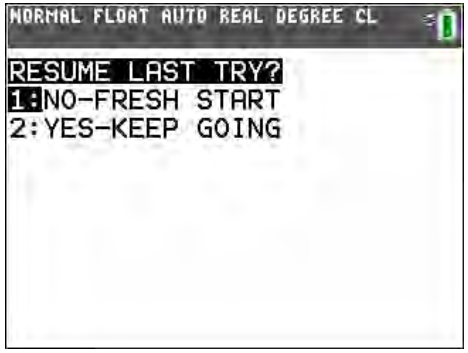


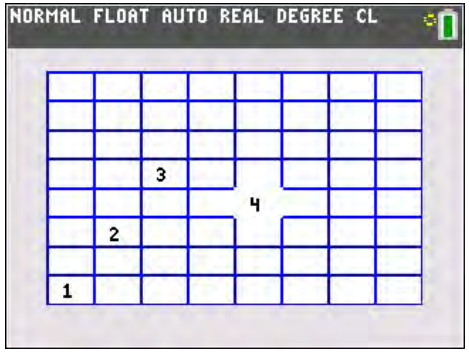












```
001 Lbl I1
002 ClrHome
003 Menu("THE KNIGHTS TOUR","MAKE ONE",I2,"INFORMATION",I3,"CREATED BY",I4)
004 Lbl I3
005 ClrHome
006 Output(1,1,"PRESS  THE CLEAR")
007 Output(2,1,"KEY TO  GO BACK.")
008 Output(3,2,"PRESS THE  2ND")
009 Output(4,1,"KEY TO  EXIT AND")
010 Output(5,1,"SAVE")
011 Output(8,4,"<PRESS ENTER>")
012 Pause
013 Goto I1
014
015 Lbl I4
016 ClrHome
017 Output(1,4,"CREATED  BY")
018 Output(2,2,"RON  LANCASTER")
019 Output(4,1,"RON2718▫NAS.NET")
020 Output(8,4,"<PRESS ENTER>")
021 Pause
022 Goto I1
023
024 Lbl I2
025 0→N
026
027 ClrHome
028 PlotsOff
029 FnOff
030 AxesOff
```

This program was created by Ron Lancaster during two connecting flights from Toronto to Singapore in 2001.

It was written entirely in the air on a TI-83 Plus. Revisions were recently done for the TI-84 Plus CE to ensure the text is displayed properly on the larger screen.

```
031 ClrDraw
032
033 Ø→Xmin
034 24→Xmax
035 Ø→Ymin
036 16→Ymax
037
038 Ø→X:Ø→Q
039 Ø→W:Ø→Z
040
041 ClrList ιCELL,ιSTATE,ιPATH,ιXCOOR,ιYCOOR,ιXTEXT,ιYTEXT,ιROW,ιSIZE
042 SetUpEditor ιCELL,ιSTATE,ιPATH,ιXCOOR,ιYCOOR,ιXTEXT,ιYTEXT,ιROW,ιFILE1,ιFILE2,ιFILE3,ιFILE4,ιFILE5,ιFILE6,ιSIZE
043 {1,8,16,24,32,4Ø,47,55}→ιROW
044
045 If dim(ιFILE1)=Ø
046 Goto C
047
048 Menu("RESUME LAST TRY?","NO-FRESH START",A,"YES-KEEP GOING",B)
049 Lbl A
050 Goto C
051
052 Lbl B
053
054 ιFILE7→ιSIZE
055 ιSIZE(1)→S
056 ιSIZE(2)→T
057
058 seq(X,X,1,ST,1)→ιCELL
059 ST→dim(ιSTATE)
060
```

```
061 ιFILE1→ιSTATE
062 ιFILE2→ιPATH
063 ιFILE3→ιXCOOR
064 ιFILE4→ιYCOOR
065 ιFILE5→ιXTEXT
066 ιFILE6→ιYTEXT
067
068 dim(ιPATH)→N
069 ιXCOOR(N)→X
070 ιYCOOR(N)→Q
071 X→W
072 Q→Z
073
074 For(I,1,dim(ιPATH),1)
075 Text(ιXTEXT(I),ιYTEXT(I),I)
076 End
077
078 For(I,0,2S,2)
079 Line(0,I,3T,I)
080 End
081 For(I,0,3T,3)
082 Line(I,2S,I,0)
083 End
084
085 Goto 1
086
087 Lbl C
088
089 Menu("CHESSBOARD SIZE?","3 ROWS",R3,"4 ROWS",R4,"5 ROWS",R5,"6 ROWS",R6,"7 ROWS",R7,"8 ROWS",R8)
090 Lbl R3
```

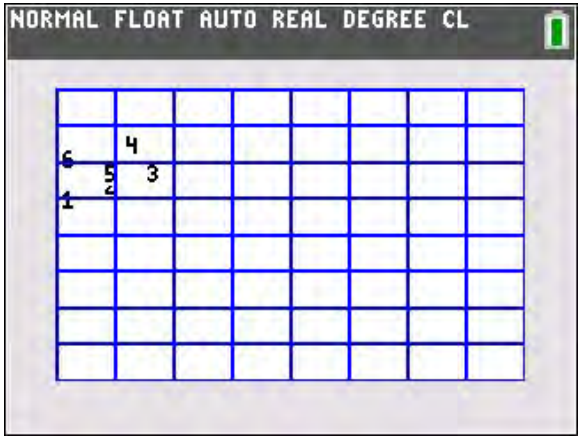
```
091 3→S
092 Goto D
093 Lbl R4
094 4→S
095 Goto D
096 Lbl R5
097 5→S
098 Goto D
099 Lbl R6
100 6→S
101 Goto D
102 Lbl R7
103 7→S
104 Goto D
105 Lbl R8
106 8→S
107 Goto D
108
109 Lbl D
110
111 Menu("CHESSBOARD SIZE?","3 COLUMNS",C3,"4 COLUMNS",C4,"5 COLUMNS",C5,"6 COLUMNS",C6,"7 COLUMNS",C7,"8 COLUMNS",C8)
112 Lbl C3
113 3→T
114 Goto E
115 Lbl C4
116 4→T
117 Goto E
118 Lbl C5
119 5→T
120 Goto E
```

```
Lbl C6
6→T
Goto E
Lbl C7
7→T
Goto E
Lbl C8
8→T
Goto E

Lbl E
S→ιSIZE(1)
T→ιSIZE(2)
seq(X,X,1,ST,1)→ιCELL
ST→dim(ιSTATE)

For(I,Ø,2S,2)
Line(Ø,I,3T,I)
End

For(I,Ø,3T,3)
Line(I,2S,I,Ø)
End

StorePic 1

Lbl 1
3X→A
2Q→B
3X+3→C
```

```
151 2Q→D
152 3X+3→E
153 2Q+2→F
154 3X→G
155 2Q+2→H
156
157 1→I
158
159 Lbl 2
160
161 getKey→M
162
163 -1*I→I
164 (I>0)→J
165 Line(A,B,C,D,J)
166 Line(C,D,E,F,J)
167 Line(E,F,G,H,J)
168 Line(G,H,A,B,J)
169
170 If M=25
171 Then
172 If Q+1>S-1
173 Goto 2
174 Q+1→Q
175 RecallPic Pic1
176 Goto 1
177 End
178
179 If M=24
180 Then
```

```
181 If X-1<Ø
182 Goto 2
183 X-1→X
184 RecallPic Pic1
185 Goto 1
186 End
187
188 If M=26
189 Then
19Ø If X+1>T-1
191 Goto 2
192 X+1→X
193 RecallPic Pic1
194 Goto 1
195 End
196
197 If M=34
198 Then
199 If Q-1<Ø
2ØØ Goto 2
2Ø1 Q-1→Q
2Ø2 RecallPic Pic1
2Ø3 Goto 1
2Ø4 End
2Ø5
2Ø6 If M=1Ø5
2Ø7 Then
2Ø8 ST-T+1+X-TQ→U
2Ø9 (W-X)²+(Z-Q)²→V
21Ø If ⌊STATE(U)=1
```

```
211 Goto 2
212 If V=5 or N=Ø
213 Then
214 N+1→N
215 U→ʟPATH(N)
216 1→ʟSTATE(U)
217 X→ʟXCOOR(N)
218 Q→ʟYCOOR(N)
219 X→W
22Ø Q→Z
221 ʟROW(8-Q)→ʟXTEXT(N)
222 4+12X-2(N>9)→ʟYTEXT(N)
223
224 int((142/54)(ʟXTEXT(N)-1)+5)→ʟXTEXT(N)
225 int((23Ø/84)(ʟYTEXT(N)-3)+1Ø)→ʟYTEXT(N
226
227 Text(ʟXTEXT(N),ʟYTEXT(N),N)
228
229 If N=ST
23Ø Then
231 StorePic 2
232 ClrHome
233 Output(1,1,"CONGRATULATIONS!")
234 Output(3,1,"AN IMAGE OF YOUR")
235 Output(4,2,"TOUR HAS  BEEN")
236 Output(5,1,"STORED  IN Pic2.")
237 Output(6,2,"PRESS ENTER TO")
238 Output(7,1,"STORE THE  STEPS")
239 Output(8,3,"OF YOUR TOUR")
24Ø Pause
```



This is what the output looked like when the original program for the TI-83 Plus was run on the new TI-84 Plus CE. All the numbers appeared out of place and were in the upper left corner. The code in lines 224 and 225 adjusts the coordinates for the text so that the numbers are displayed in their correct cells.

- Plus Edition: 96 (0-95) × 64 (0-63) pixels, monochrome (software grayscale can be used) LCD
- Plus C/CE: 320 × 240-pixel screen, 140 DPI, 16-bit color. Drawing pixel range: 0-164 x 0-264.

```
241 Goto 6A
242 End
243
244 End
245 End
246
247 If M=45 and sum(ιSTATE)≠Ø
248 Then
249 Text(ιXTEXT(N),ιYTEXT(N),"        ")
250 RecallPic Pic1
251 ιXCOOR(N)→X
252 ιYCOOR(N)→Q
253 X→W
254 Q→Z
255 ST-T+1+X-TQ→U
256 Ø→ιSTATE(U)
257 N-1→N
258 N→dim(ιPATH)
259 N→dim(ιXCOOR)
260 N→dim(ιYCOOR)
261 N→dim(ιXTEXT)
262 N→dim(ιYTEXT)
263
264 If N=Ø
265 Goto 1
266
267 ιXCOOR(N)→X
268 ιYCOOR(N)→Q
269 X→W
270 Q→Z
```

```
271 Goto 1
272 End
273
274 If M=21
275 Then
276 ClrHome
277 Menu("STOP THE TOUR?","YES",4,"NO",5)
278 Lbl 5
279 Goto 2
280 Lbl 4
281 ClrHome
282
283 If N=0
284 Goto 7
285
286 Lbl 6A
287 ClrHome
288 Menu("SAVE YOUR TOUR?","YES",6,"NO",7)
289 Lbl 6
290
291 ιSTATE→ιFILE1
292 ιPATH→ιFILE2
293 ιXCOOR→ιFILE3
294 ιYCOOR→ιFILE4
295 ιXTEXT→ιFILE5
296 ιYTEXT→ιFILE6
297 ιSIZE→ιFILE7
298
299 Goto 8
300
```

```
301 Lbl 7
302 Ø→dim(ιFILE1)
303 Ø→dim(ιFILE2)
304 Ø→dim(ιFILE3)
305 Ø→dim(ιFILE4)
306 Ø→dim(ιFILE5)
307 Ø→dim(ιFILE6)
308
309 Lbl 8
310 ClrHome
311 Output(1,1,"SUMMARY:")
312 Output(2,1,"TOTAL NUMBER OF")
313 Output(3,1,"SQUARES VISITED")
314 Output(4,1,"=")
315 Output(4,3,N)
316 Output(8,4,"<PRESS ENTER>")
317 Pause
318 ClrHome
319 ClrDraw
320 SetUpEditor
321 Stop
322 End
323
324 Goto 2
325
326
327
328
329
330
```

Magic squares that will take your breath away...

This 9 x 9 square is additive and multiplicative. The **sum** of the 9 numbers in every row, column and main diagonal is the same. The **product** of the 9 numbers in every row, column and main diagonal is the same.

What strategies would you use to make another one of these additive and multiplicative squares?

17	171	126	54	230	100	93	264	145
124	66	290	85	57	168	162	23	225
216	115	75	279	198	29	170	76	42
261	186	33	210	68	38	200	135	69
50	270	92	87	248	165	21	153	114
105	51	152	150	27	207	116	62	330
138	25	243	132	58	310	95	63	136
190	84	34	184	125	81	297	174	31
99	232	155	19	189	102	46	250	108

Created by Gakuho Abe from Akita-ken, Japan in 1962.
Source: Mathematics on Vacation, Joseph Madachy

magic sum = 1,200
magic product = 1,619,541,385,529,760,000

1111	8881	8818	1188
8188	1818	1881	8111
1888	8118	8181	1811
8811	1181	1118	8888

NOON ZOONOOZ

Bordered magic square created by Antoine Arnaud (1616 – 1698)
10 x 10, 8 x 8, 6 x 6, 4 x 4 magic squares!

11	99	50	4	96	95	7	10	92	41
1	12	88	14	86	85	17	83	19	100
98	49	33	77	48	28	74	43	52	3
21	22	23	64	36	35	67	78	79	80
70	69	76	57	45	46	54	25	32	31
30	39	75	47	55	56	44	26	62	71
81	72	38	34	66	65	37	63	29	20
93	59	58	24	53	73	27	68	42	8
40	82	13	87	15	16	84	18	89	61
60	2	51	97	5	6	94	91	9	90

The magic of Lee Sallows

5	22	18
28	15	2
12	8	25

The magic of Lee Sallows

5	22	18
28	15	2
12	8	25

five	twenty-two	eighteen
twenty-eight	fifteen	two
twelve	eight	twenty-five

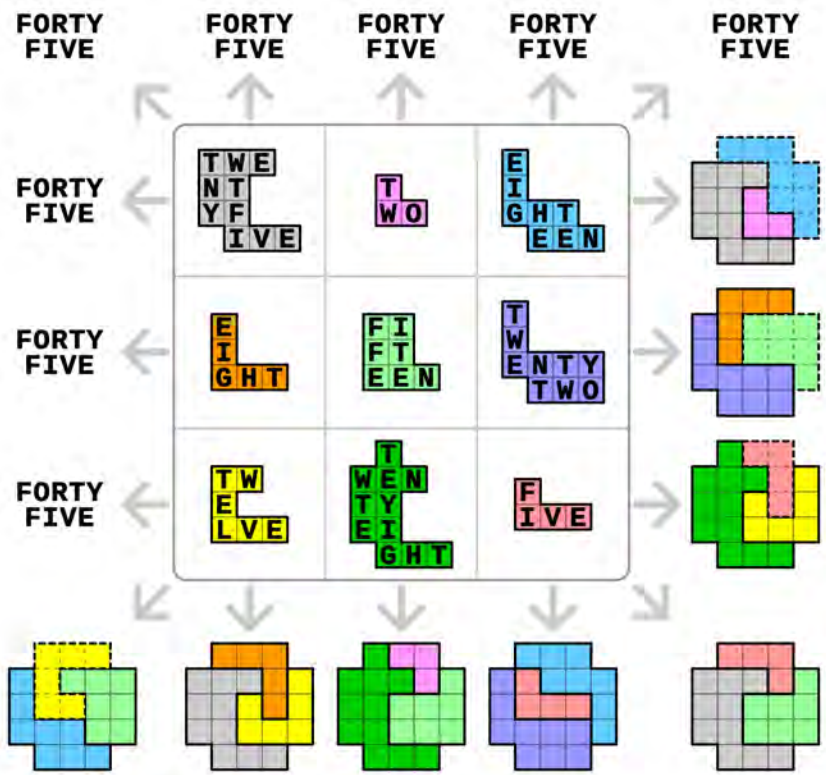
The magic of Lee Sallows

5	22	18
28	15	2
12	8	25

five	twenty-two	eighteen
twenty-eight	fifteen	two
twelve	eight	twenty-five

4	9	8
11	7	3
6	5	10

Geometric magic squares created by Lee Sallows
https://en.wikipedia.org/wiki/Geometric_magic_square



Stamps issued by the People's Republic of China
https://www.theguardian.com/science/alexs-adventures-in-numberland/2014/nov/04/macaus-magic-square-stamps-just-made-philately-even-more-nerdy

To download files from this presentation, go to bit.ly/MAGICOFTI

Books

Benjamin Franklin's Numbers: An Unsung Mathematical Odyssey by Paul C. Pasles

Geometric Magic Squares: A Challenging New Twist Using Colored Shapes Instead of Numbers by Lee Sallows

Magic Squares and Cubes by W. S. Andrews

New Recreations with Magic Squares by William H. Benson & Oswald Jacoby

The Wonders of Magic Squares by Jim Moran

The Zen of Magic Squares, Circles, and Stars by Clifford Pickover

Websites

Magic Squares, Magic Stars & Other Patterns
http://recmath.org/Magic Squares/

Magic Square Lexicon by H. D. Heinz & J. R. Hendricks
http://recmath.org/Magic Squares/Downloads/Lexicon_Sample.pdf

Articles

Discovering the Magic of Magic Squares
Ingrid Semanišinová and Marián Trenkler
Mathematics Teacher, 101(1), 32-39.

Magic Squares: Discovering Their History and Their Magic
Dawn L. Anderson
Mathematics Teaching in the Middle School, 6(8), 466-471

The Magic of Balanced Groups: Educational Applications of Magic Squares
Michael J. Bosse, N. R. Nandakumar and Melanie L. Ore
Mathematics Teacher, 100(5) 329-337

The Most Magical of All Magic Squares
Frank J. Swetz
Mathematics Teacher, 94(6), 458-63

The Versatile Magic Square
Gale A. Watson, Gale A.
Mathematics Teaching in the Middle School, 8(5), 252-55