TI in Focus: AP® Calculus

2022 AP® Calculus Exam: AB1/BC1
Scoring Guidelines and Solutions

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Outline

(1) Free Response Question

(2) Point Distribution

(3) Solutions (using technology)

(4) Scoring Notes

(5) Common Errors

(6) Problem Extensions

Part A (AB or BC): Graphing calculator required

Question 1 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

	Model Solution	Scoring	
(a)	Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).		
	The total number of vehicles that arrive at the toll plaza from 6 A.M. to 10 A.M. is given by $\int_1^5 A(t)\,dt$.	Answer	**1 point**

Solution

$$\int_1^5 A(t)\,dt$$

$$\int_1^5 450\sqrt{\sin(0.62t)}\,dt$$

Scoring notes:

- The response must be a definite integral with correct lower and upper limits to earn this point.

$\int_0^5 A(t)\,dt$	0
$\int A(t)\,dt$	0

- Because $|A(t)| = A(t)$ for $1 \le t \le 5$, a response of $\int_1^5 \left|450\sqrt{\sin(0.62t)}\right| dt$ or $\int_1^5 |A(t)|\,dt$ earns the point.

- A response missing dt or using dx is eligible to earn the point.

$$\int_1^5 A(t)$$

$$\int_1^5 A(t)\,dx$$

- A response with a copy error in the expression for $A(t)$ will earn the point only in the presence of $\int_1^5 A(t)\,dt$.

$$\int_1^5 A(t)\,dt = \int_1^5 400\sqrt{\sin(0.62t)}\,dt$$ 1

$$\int_1^5 400\sqrt{\sin(0.62t)}\,dt$$ 0

Common Errors

(1) Correct conceptual idea, but lack of notational fluency.

$$A(t) = \int_1^5 A(t)\,dt$$ 0

(2) Inclusion of a constant with the integral or the integrand.
This did not earn the point if the constant was ambiguous or clearly nonzero.

$$\int_1^5 A(t)\,dt + 4$$ 0

$$\int_1^5 A(t) + 4$$ 0

(3) Inclusion of a summation symbol.

$$\sum \int_1^5 A(t)\,dt$$ 0

Common Errors (Continued)

(4) Other examples of expressions that did not earn the point.

$$\int_1^5 A'(t)\,dt$$

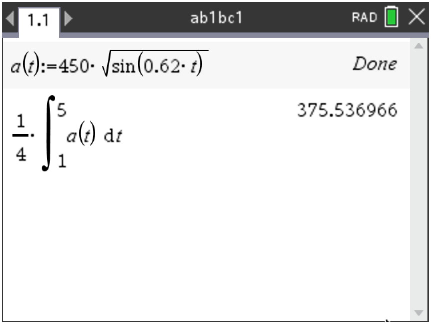
$$\int_6^{10} A(t)\,dt$$

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

Average $= \dfrac{1}{5-1}\displaystyle\int_1^5 A(t)\,dt = 375.536966$	Uses average value formula: $\dfrac{1}{b-a}\displaystyle\int_a^b A(t)\,dt$ **1 point**
The average rate at which vehicles arrive at the toll plaza from 6 A.M. to 10 A.M. is 375.537 (or 375.536) vehicles per hour.	Answer **1 point**

Solution

$$\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.537$$



Scoring notes:

- The use of the average value formula, indicating that $a = 1$ and $b = 5$, can be presented in single or multiple steps to earn the first point. For example, the following response earns both points: $\int_1^5 A(t)\,dt = 1502.147865$, so the average value is 375.536966.

Cannot use incorrect notation to connect these two values.

- A response that presents a correct integral along with the correct average value, but provides incorrect or incomplete communication, earns 1 out of 2 points. For example, the following response earns 1 out of 2 points: $\int_1^5 A(t)\,dt = 1502.147865 = 375.536966$.

This is an example of incorrect notation.

- The answer must be correct to three decimal places. For example, $\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.536966 \approx 376$ earns only the first point.

$$\frac{1}{5-1}\int_1^5 A(t)\,dt = 375.54$$ 1 - 0

- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $\frac{1}{4}\int_1^5 A(t)\,dt = 79.416068$.

- Special case: $\frac{1}{5}\int_1^5 A(t)\,dt = 300.429573$ earns 1 out of 2 points.

Common Errors

(1) Average rate of change of A instead of average value of A.

$$\frac{A(5) - A(1)}{5 - 1}$$

(2) Use of $A'(T)$ as the integrand: $\displaystyle \frac{1}{5-1}\int_1^5 A'(t)\,dt$

(3) Rounding the average value to an integer: 375 or 376

(4) Arithmetic errors: $\displaystyle \frac{1}{5-1}\int_1^5 A(t)\,dt = \frac{1}{5}\int_1^5 A(t)\,dt$

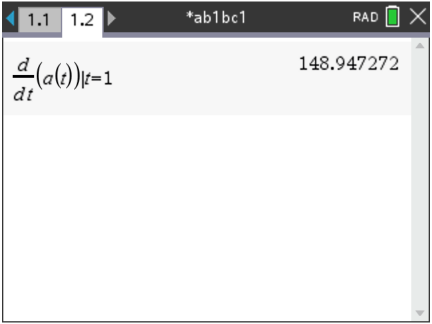
(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

$A'(1) = 148.947272$	Considers $A'(1)$	**1 point**
Because $A'(1) > 0$, the rate at which the vehicles arrive at the toll plaza is increasing.	Answer with reason	**1 point**

Solution

$$A'(1) = 148.947272$$

The rate at which vehicles arrive at the toll booth plaza is increasing because $A'(1) > 0$.



Scoring notes:

- The response need not present the value of $A'(1)$. The second line of the model solution earns both points.

$A'(1) > 0$.
Therefore the rate at which the vehicles arrive at the toll booth plaza is increasing.

- An incorrect value assigned to $A'(1)$ earns the first point (but will not earn the second point).

$A'(1) = 150$ 1-0

- Without a reference to $t = 1$, the first point is earned by any of the following:
  - 148.947 accurate to the number of decimals presented, with zero up to three decimal places (i.e., 149, 148, 148.9, 148.95, or 148.94)
  - $A'(t) = 148.947$ by itself

- To be eligible for the second point, the first point must be earned.

0 - 1 not possible.

- To earn the second point, there must be a reference to $t = 1$.

- Degree mode: $A'(1) = 23.404311$

Common Errors

(1) Inability to communicate precisely what was needed to earn both points. A one sentence answer here could earn both points.

(2) Use of Leibniz notation: $A'(t)$, $\dfrac{dA}{dt}$

(3) Discussion involving $A'(t)$, not about $A'(1)$.

(4) Use of $A'(6)$ (6 AM), not penalized for the same error in (a) of (b).

(5) Use of the phrase *the derivative* instead of *the derivative of* A.

(6) Use of the phrase *the rate of* $A(t)$ instead of *the rate of change of* $A(t)$.

(7) Symbolic differentiation: if incorrect then not eligible for the second point.

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by $N(t) = \int_a^t (A(x) - 400)\,dx$, where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

$N'(t) = A(t) - 400 = 0$

$\Rightarrow A(t) = 400 \Rightarrow t = 1.469372,\ t = 3.597713$

$a = 1.469372$

$b = 3.597713$

Considers $N'(t) = 0$ — **1 point**

$t = a$ and $t = b$ — **1 point**

t	$N(t) = \int_a^t (A(x) - 400)\,dx$
a	0
b	71.254129
4	62.338346

The greatest number of vehicles in line is 71.

Answer — **1 point**

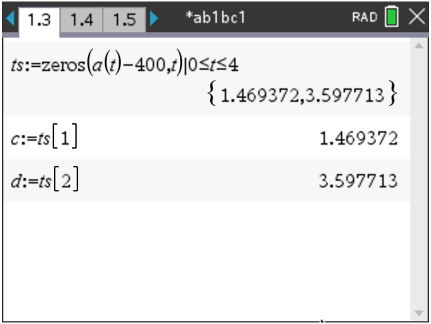
Justification — **1 point**

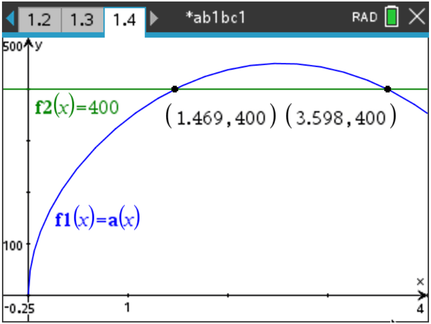
Solution

Find the absolute maximum value of the function $N(t) = \int_a^t (A(x) - 400)\,dx$

over the interval $a \leq t \leq 4$

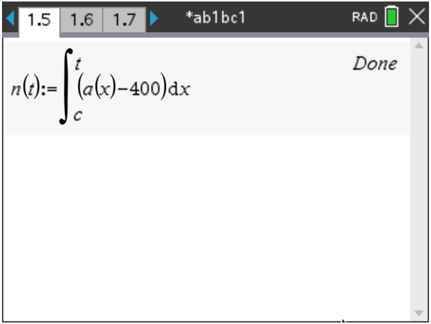
$$N'(t) = A(t) - 400 = 0 \quad \Rightarrow \quad t = 1.469372,\ t = 3.597713$$

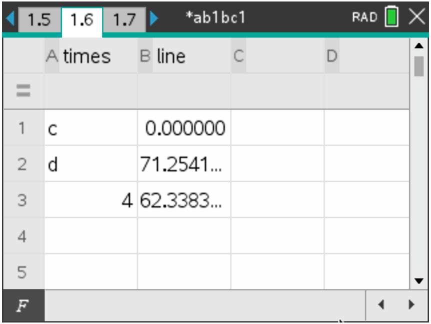




Solution (Continued)

Consider a table of values.





The greatest number of vehicles in line is 71.

Scoring notes:

- It is not necessary to indicate that $A(t) = 400$ to earn the first point, although this statement alone would earn the first point.

This statement is implied in the presence of $t = a$ and/or $t = b$.

- A response of "$A(t) \geq 400$ when $1.469372 \leq t \leq 3.597713$" will earn the first 2 points. A response of "$A(t) \geq 400$" along with the presence of exactly one of the two numbers above will earn the first point, but not the second. A response of "$A(t) \geq 400$" by itself will not earn either of the first 2 points.

- To earn the second point the values for a and b must be accurate to the number of decimals presented, with at least one and up to three decimal places. These may appear only in a candidates table, as limits of integration, or on a number line.

$t = 1.4, 3.5, \quad t = 1.46,\ 3.59, \quad t = 1.469,\ 3.597$

- A response with incorrect notation involving t or x is eligible to earn all 4 points.

- A response that does not earn the first point is still eligible for the remaining 3 points.

- To earn the third point, a response must present the greatest number of vehicles. This point is earned for answers of either 71 or 71.254*** only.

Either the nearest whole number or with three digits to the right of the decimal.

- A correct justification earns the fourth point, even if the third point is not earned because of a decimal presentation error.

- When using a Candidates Test, the response must include the values for $N(a)$, $N(b)$, and $N(4)$ to earn the fourth point. These values must be correct to the number of decimals presented, with up to three decimal places. (Correctly rounded integer values are acceptable.)

The response must include these three values.
The values must be correct, to the number of decimals presented, up to three.

- Alternate solution for the third and fourth points:

  For $a \le t \le b$, $A(t) \ge 400$. For $b \le t \le 4$, $A(t) \le 400$.

  Thus, $N(t) = \int_a^t (A(x) - 400)\,dx$ is greatest at $t = b$.

  $N(b) = 71.254129$, and the greatest number of vehicles in line is 71.

- Degree mode: The response is only eligible to earn the first point because in degree mode $A(t) < 400$.

Common Errors

(1) Incomplete table of values, or only justification for a local maximum.

(2) No value for $N(a)$ in a table of values.

(3) Presentation of a graph indicating the intersection of the graph of A and the horizontal line $y = 400$. This did not earn the first point, but was eligible for the other points.

(4) Sign charts: not sufficient justification.

(5) $N'(t)$ changes sign at $t = 3.598$: local argument.

Additional Questions

(1) Find the minimum rate at which vehicles arrive at the toll plaza from 6 AM ($t = 1$) to 10 AM ($t = 5$).

(2) From 5 AM to 10 AM, the rate at which vehicles leave the toll plaza is given by $L(t) = 150 \ln(t + 1)$, where t is the number of hours after 5 AM and $L(t)$ is measured in vehicles per hour. Find the number of cars in line at the toll plaza at 8 AM.

(3) Find the time at which the number of cars in line begins to decrease.

(4) If the number of vehicles in line reaches 500, then another toll booth is opened at the plaza. Find the first time t, $0 \leq t \leq 5$, for which this happens.

(5) Find the maximum number of cars in line at the toll plaza for $0 \leq t \leq 5$.

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