Building Concepts: Connecting Ratios to Equations

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson allows students to understand that a proportional relationship is a collection of equivalent ratios. A collection of equivalent ratios, *cA*:*cB*, for any positive number *c*, can be graphed in the coordinate plane using the coordinates (*cA*, *cB*). The graph represents a proportional relationship and lies on a ray with the endpoint at the origin.

A proportional relationship can be described by an equation of the form $y = kx$, where k is a positive constant, often called a *constant of proportionality*.

Learning Goals

1. Recognize that proportional relationships involve collections of equivalent ratios;
2. associate a unit rate with the coordinate pair with first coordinate 1;
3. recognize that the members of any set of equivalent ratios have the same unit rate;
4. graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope;
5. identify the constant of proportionality, k, as the unit rate or the slope of a line through the origin;
6. connect ratios to equations of a line through the origin: $a:b \rightarrow y = \frac{b}{a}x$.

Prerequisite Knowledge

Connecting Ratios to Equations is the tenth lesson in a series of lessons that explore the concepts of ratios and proportional relationships. The lesson builds on students' prior knowledge connecting ratios to graphs. Prior to working on this lesson, students should have completed *Ratios and Rational Numbers* and *Connecting Ratios to Graphs.* Students should understand:

- how to plot points in a coordinate grid by associating a ratio with an ordered pair of values;
- how to generate a table of equivalent ratios.

Vocabulary

- **proportional relationship**: a collection of pairs of numbers that are in equivalent ratios.
- **constant of proportionality**: the value $\frac{B}{A}$ for the ratio A:B.
- **slope triangle**: a right triangle whose legs are the horizontal and vertical changes moving from point to point on a line and whose hypotenuse is a segment of the line.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Connecting Ratios to Equations_Student.pdf
- Connecting Ratios to Equations_Student.doc
- Connecting Ratios to Equations.tns
- Connecting Ratios to Equations_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

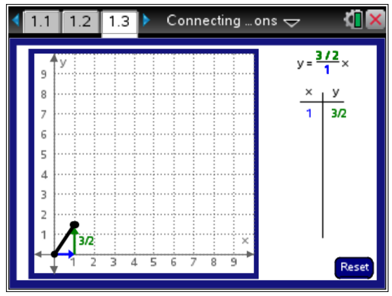
This TI-Nspire™ lesson allows students to understand that a proportional relationship is a collection of equivalent ratios. A collection of equivalent ratios, $cA{:}cB$, for any positive number c, can be graphed in the coordinate plane using the coordinates (cA, cB). The graph represents a proportional relationship and lies on a ray with the endpoint at the origin. This lesson focuses on the fact that a proportional relationship can be described by an equation of the form $y = kx$, where k is a positive constant, often called a *constant of proportionality*. For a ratio A:B, the constant of proportionality, k, is equal to the value $\frac{B}{A}$. The rate of change in y for a given unit in x will be constant for the points on the line. A unit rate associated with the set of ratios is the amount of increase in y for every one unit increase in x and can be visualized in the graph of the line as the vertical increase in a "slope triangle" with horizontal side one unit in length. The unit rate appears in the equation of a proportional relationship as the coefficient of x (often in the form $\frac{B}{A}$ rather than $\frac{(B/A)}{1}$), in the graph as a measure of the steepness or the slope of the line, and in the coordinate pair with first coordinate 1. For a given coordinate axes, the greater the (positive) unit rate or "slope," the steeper the line.

The terms describing proportional relationships can be confusing: The ratio A:B is associated with a collection of equivalent ratios, cA:cB. These ratios are associated with coordinate pairs (cA, cB) that graph as a straight line through the origin. The unit rate for this set of equivalent ratios is the second number in the ordered pair (1, $\frac{B}{A}$); the slope of the line is the constant of proportionality $\frac{B}{A}$.

Part 1, Page 1.3

Focus: How are ratios, slope of a line, constant of proportionality and unit rates connected to equations of lines?

On page 1.3, the dot on the graph can be moved to create a segment from (0, 0) to that point with blue horizontal and green vertical arrows indicating the change in x and the change in *y, respectively.* Selecting the horizontal arrow twice will generate another point and extend the segment. Selecting each successive horizontal arrow twice continues to extend the line and display slope triangles as well as record the coordinates of the points in the table.



TI-Nspire Technology Tips

Set the rate by dragging the endpoint of the segment.

Use the arrow keys to move the endpoint of the segment. Press enter to set the slope of the line.

Reset returns to the original screen or press ctrl del on the handheld to reset.

The following questions have students generate slope triangles from the graph of a collection of equivalent ratios, thus leading to the notion of a proportional relationship.

Class Discussion

Have students…	Look for/Listen for…
Look at page 1.3.	
• ***Describe how the table and graph seem to be related.***	Answer: The horizontal arrow points to the location 1 unit from the origin. The vertical arrow points to the location $\frac{3}{2}$ from the origin, which gives the point $(1, \frac{3}{2})$ that is plotted on the graph and shown in the table.
• ***What set of equivalent ratios could be represented in the graph and the table?***	Answer: Any ratio that is equivalent to $1:\frac{3}{2}$.

Class Discussion (continued)

Have students...	Look for/Listen for…
Generate another horizontal and vertical line segment.	
• ***What stays the same and what changes each time you generate a triangle?***	Answer: Each selection generates 1) a new point on the line using a triangle, which is always the same: over 1 unit to the right and up $\frac{3}{2}$ units; 2) a new row in the table; and 3) the equation does not change.
• ***How are the points on the graph and the values in the table related?***	Answer: The values in the table are the points on the graph for each "over to the right 1 and up $\frac{3}{2}$" starting from (0, 0).
• ***Do the values in the table and the points on the line support your answer to the question above with respect to the set of equivalent ratios represented in the table and graph? Why or why not?***	Answer: Yes, all of the points represent ratios equivalent to 1: $\frac{3}{2}$ because they are all the product of a positive number and the values 1 and $\frac{3}{2}$.

A proportional relationship is a collection of equivalent ratios. The table below can be used to generate a proportional relationship using ratios equivalent to 1: $\frac{3}{2}$.

- ***Fill in the table. You may want to use the horizontal and vertical arrows to help your thinking.***

Answer:

X	*Number of* $\frac{3}{2}$ *'s*	*y*
1	$1 \times \frac{3}{2}$	$\frac{3}{2}$
2	$2 \times \frac{3}{2}$	$\frac{6}{2} = 3$
3		
4		
5		
6		

X	*Number of* $\frac{3}{2}$ *'s*	*y*
1	$1 \times \frac{3}{2}$	$\frac{3}{2}$
2	$2 \times \frac{3}{2}$	$\frac{6}{2} = 3$
3	$3 \times \frac{3}{2}$	$\frac{9}{2}$
4	$4 \times \frac{3}{2}$	$\frac{12}{2} = 4$
5	$5 \times \frac{3}{2}$	$\frac{15}{2}$
6	$6 \times \frac{3}{2}$	$\frac{18}{2} = 9$

- ***How are the entries in the table related to the graph?***

  Answer: The *x*- and the *y*-values are coordinates of points on the line.

Entries in a table can often be predicted by looking for relationships or patterns.

- ***Identify a rule that enables you to fill in the next row of the table from the row above without selecting the horizontal arrow.***

  Answer: Each *x*-value increases by 1, and each *y*-value increases by $\frac{3}{2}$.

- ***What is the rate of the change in the y-value for each 1-unit change in the x-value?***

  Answer: $\frac{3}{2}$ change in every *y*-value per a 1-unit change in the *x*-values.

- ***What fraction can be associated with your answer to the previous question?***

  Answer: $\frac{3/2}{1}$ or $\frac{3}{2}$.

Student Activity Questions—Activity 1

In the following questions, a proportional relationship is formally associated with an equation, and the slope of the line representing the proportional relationship is associated with the notion of unit rate. Students compare the slopes of two lines and make conjectures about the relationship between the steepness of the slopes and the value of value of the slope. You may want to take more than one session in order to cover both parts of the activity.

1. **The ratio of the change in each *y*-value to a 1-unit change in the *x*-value for a graph of a proportional relationship is associated with a unit rate, the vertical increase in a "unit rate triangle" or "slope triangle" with horizontal side of length 1. The value is called the rate of change or the slope of the line.**

   a. **On page 1.3, describe a slope triangle and give the coordinates of the three points that determine the triangle.**

   Possible answer: (0, 0), (1, 0), and (1, $\frac{3}{2}$)

   b. **Why do you think the triangle is called a "slope" triangle?**

   Possible answer: The legs of the slope triangle indicate the horizontal and vertical change between two points on a line, which is the hypotenuse of the triangle. The slope triangle measures the steepness or the slope of the line.

2. **General statements about relationships between quantities can often be expressed using symbols. An equation is given above the table on page 1.3.**

   a. **How is this equation related to the table and the graph?**

   Answer: The equation $y = \frac{3/2}{1}x$ says that every *y*-value in the table can be generated by the product of the *x*-value and the unit rate. The equation describes the points on the line formed by the ratios equivalent to 1: $\frac{3}{2}$.

   b. **The rate of change (slope) of the line representing a proportional relationship is called the constant of proportionality. What is the constant of proportionality for the line on page 1.3?**

   Answer: $\frac{3/2}{1}$ or $\frac{3}{2}$.

3. **Reset page 1.3. Move the point over 1 and up 3.**

   a. **What is the unit rate?**

   Answer: $\frac{3}{1}$ or 3.

   b. **Use the unit rate to predict the next two points on the graph and in the table. Check your prediction using the TNS lesson**.

   Answer: (2, 6) and (3, 9).

Additional Discussion

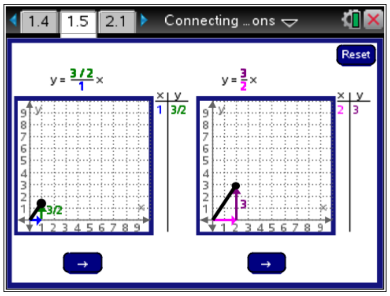
Have students…	Look for/Listen for…
Which of the following do you think is true? Explain your thinking.	
a. The unit rate for a given line can be either $\frac{1}{c}$ ***or*** $\frac{c}{1}$ ***if*** $c \neq 0$. ***b. Every y-value in the table is a product of the unit rate and an x-coordinate.*** ***c. The unit rate for a straight line will be constant.*** ***d. Every point on a line belongs to a set of ratios that are equivalent to the ratio formed by the unit rate for the line to 1.***	Answer: a) is false because the way a unit rate is defined, the horizontal change has to be 1. The other three are true. See the table for b). c) is true because if the unit rate was not constant, the line would go up or down more or less for a 1-unit change, and then it would not be a straight line. d) is true because a set of equivalent ratios is associated with points that lie on a straight line through the origin where a unit rate can be used to generate a subset of those points.
Which of the following points is associated with a unit rate? Explain why or why not in the context of a slope triangle. You might want to use the TNS lesson to help your thinking.	
a. $(\frac{5}{2}, 1)$ ***b.*** $(1, \frac{5}{2})$ ***c. (1, 1)*** ***d. (5, 2)***	Answer: a) No because this would produce a slope triangle of over $\frac{5}{2}$ and up 1; a unit rate is the reverse, over 1 and up some quantity. b) Yes, this would produce a slope triangle of over 1 unit and up $\frac{5}{2}$ units. c) Yes, this would produce a slope triangle of over 1 and up 1. d) No, this would produce a slope triangle of over 5 and up 2, not in terms of over 1 and up some quantity.
• ***If Sabra runs 5 meters every 2 seconds, give the unit rate for her pace and interpret it in terms of the context. Explain how you found your answer.***	Answer: $\frac{5/2}{1}$ or $\frac{5}{2}$ up for every 1 unit over. Sabra would go $\frac{5}{2}$ or 2.5 meters every second. You can divide the numerator of the fraction associated with the ratio $\frac{5}{2}$ by its denominator, 2.

Additional Discussion (continued)

Have students…	**Look for/Listen for…**
• ***Write an equation describing how far she runs in terms of time. Be sure to explain what the elements of the equation represent.***	Answer: If t is time in seconds and d is distance in meters, $d = \frac{5}{2} \times t$. Note that the units make sense, d meters $= \frac{5}{2} \times \frac{m}{\text{sec}} \times \text{sec}$.
• ***Give an example of how you might use the equation in a problem.***	Possible answer: If Sabra runs at the same pace, how far will she run in 10 seconds? Be sure that students recognize that “at the same pace” means constant rate.

Part 2, Page 1.5

Focus: Students use parallel grids to compare the tables, graphs, and equations for different unit rates.



There are 2 graphs on the screen. Use the tab key to toggle between the two sides of the graph. The right/up arrows on the touchpad on the handheld change the location of the point.

Set the rate by dragging the endpoint of the segment or using the arrow keys to move the endpoint of the segment.

Press enter to set the slope of the line.

Build the line by using the right and up arrows on the keypad or on the screen.

Student Activity Questions—Activity 2

1. **Work with a partner. One of you should use the graph on the left side of page 1.5 and the other should use the graph on the right side of page 1.5. Each student should generate at least 3 points on the line and in the table.**

    a. **How do the two tables compare?**

    Answer: The tables have some of the same entries, but the table on the left side of page 1.5 has fractional entries as well as the same whole number entries.

    b. **How do the graphs compare?**

    Answer: The graphs seem to be the same line.

Student Activity Questions—Activity 2 (continued)

c. **How are the slope triangles alike and how are they different? What might explain any differences?**

Answer. The slope triangles on the left of page 1.5 have a base of 1 and a height of $\frac{3}{2}$; the slope triangles on the right of page 1.5 have a base of 2 and a height of 3. The ratios $\frac{3}{2}$:1 and 3:2 are equivalent, so although the slope triangles are different sizes, they produce the same set of points on the line. One triangle is kind of a scaled up version of the other.

d. **Compare the slope for the two equations.**

Answer: They are basically the same except on the left side of page 1.5, the multiplier of *x* is expressed as $\frac{3/2}{1}$, and on the right side of page 1.5 the multiplier is $\frac{3}{2}$. These are the same because any number divided by 1 is just that number.

2. **Reset page 1.5, and then move the point in the graph on the left side of page 1.5 to (1, 2).**

a. **With your partner, decide where to move the point in the graph on the right side of page 1.5 to a point not (1, 2) that will make the lines on each page contain the same points. Explain your reasoning**.

Answer: Any point of the form (a, 2a) because those points will be associated with ratios equivalent to 1:2.

b. **Use the TNS lesson to check your answer. How can you tell from the graphs that the lines contain the same points?**

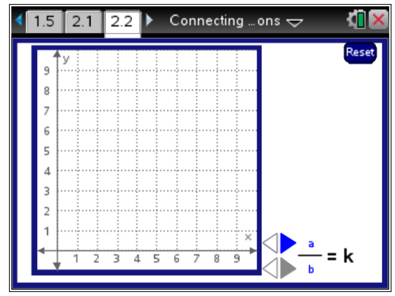
Answer: You can tell from the graph that the lines contain the same points; some of the points in the tables will also be the same. You can also tell from the equations because $y = \frac{2}{1}x$ is the same as $y = 2x$ on the left of side page 1.5. For example, $y = \frac{4}{2}x$ on the right side is the same as $y = 2x$ on left side of page 1.5, so the equations describe the same relationship.

c. **Give two other points not on either of the tables that will lie on the lines.**

Possible answer: (8,16) and (10,20).

Part 3, Page 2.2

Focus: Using a constant of proportionality to generate a line.



The arrows on page 2.2 set a constant of proportionality, ***k***, which is the slope, or rate of change, used to generate the line. Selecting **Draw** displays the line *y* = *kx*, the points on the line generated by using the unit rate and the coordinates of those points in the table.

Use the arrows to set the ratio, and then press the **Draw** button or the enter key.

> **Teacher Tip:** Have students explain how they can use the equation to predict points on the line before graphing them. Encourage them to use the TNS lesson to demonstrate their reasoning.

Class Discussion

The following question makes explicit the connection between the constant of proportionality and the slope of a proportional relationship.

Have students…	**Look for/Listen for…**
Use the arrows to set $b = 1$ and $a = 3$ to determine k. Select Draw.	
• ***How are the table, graph and equation related?***	Answer: The table displays the coordinates of points on the graph and the equation describes the relationship between the *x* and *y*-values in the proportional relationship.
• ***Identify the unit rate and the constant of proportionality.***	Answer: $\frac{1}{3}$ is the constant of proportionality, and the equivalent fraction $\frac{1/3}{1}$ is the unit rate.
• ***What point with an x-coordinate of 10 will be on the graph? Justify your answer in at least two different ways.***	Sample answer: (10, $\frac{10}{3}$) use the equation $y = \frac{1}{3}x$. For $x = 10$, the corresponding *y*-value will be $\frac{1}{3} \times 10$ or $\frac{10}{3}$ (10 copies of the unit fraction $\frac{1}{3}$). A second strategy might be to continue counting the increase of $\frac{1}{3}$ in each row of the table until you get to the 10th row.

Class Discussion (continued)

> **Teacher Tip:** The following questions review the relationship between ratios, equations, slope and the constant of proportionality.

One half gallon is equivalent to 2 quarts.

- ***Find the unit rate for the number of quarts per gallon.***
  Answer: 4 quarts per gallon

- ***Graph the line using the unit rate and the TNS lesson. Use the line to estimate the number of quarts in $2\frac{1}{2}$ gallons.***

  Answer: Reading off the graph, it would be 10 quarts.

- ***How can you check your estimate for the previous question?***

  Answer: Use the equation, $q = 4 \times g$ so 4×2.5 gallons = 10 quarts , where *q* represents the number of quarts and *g* represents the number of gallons.

- **Use at least two ways to find the number of gallons in 36 quarts.**

  Answer: You can use the equation, $36 \text{ quarts} = 4 \times g$ and solve for *g* to get 9 gallons. You can build from the table; start with the ratio 4 to 16, then 5 to 20, etc. until you get to 9 to 36. You could also think about how the ratio 4:1 has to be equivalent to *x*:9. If you multiply the second number by 9, you find that 36:9 is equivalent to *x*:9, so $x = 36$.

Student Activity Questions—Activity 3

1. **Find the equation of each proportional relationships described below. Use the TNS lesson to help your thinking**.

   a. **It contains the point (2, $\frac{3}{4}$).**

      Answer: $y = \frac{3}{8}x$

   b. **The vertical change is 3 for every 2 units of horizontal change.**

      Answer: $y = \frac{3}{2}x$

   c. **As the *x*-values in the table increase by 1, the *y*-values increase by 3.**

      Answer: $y = 3x$

Student Activity Questions—Activity 3 (continued)

2. **Given the ratio *a:b***

   a. **What is the difference between the unit rate, the slope of a line, and the constant of proportionality associated with *a:b*?**

   Answer: the ratio *a*:*b* is associated with the fraction $\frac{b/a}{1}$, which is the unit rate; the slope of the line containing the set of ratios equivalent to a:b is $\frac{b}{a}$; and the constant of proportionality is the slope of the line, $\frac{b}{a}$.

   b. **How can you find the constant of proportionality from an equation, a table, and a graph?**

   Answer: The constant of proportionality is the nonzero number *c* in the equation $y = cx$, the change in the *y*-values in the table for every unit change in the *x*-values, and the slope of the graph of a line.

Additional Discussion

Reset page 2.2, then enter $b = 4$ and $a = 3$

Have students…	Look for/Listen for…
• ***Predict what the line and the equation will be.***	Answer: The equation will be $y = \frac{4}{3}x$, and the graph will be a line generated by the unit rate over 1 and up $\frac{4}{3}$.
• ***Tami said the line could be generated by a slope triangle over 3 and up 4 as well. What would you say to Tami?***	Answer: She is correct because the ratio of 4 to 3 is equivalent to the ratio $\frac{4}{3}$ to 1 (multiply both values in $\frac{4}{3}$ to 1 by 3).
Use page 2.2 to decide whether the statements below are true. Explain your thinking in each case.	
• ***A line with a constant of proportionality of 2 is steeper than one with a constant of proportionality of $\frac{1}{2}$.***	Answer: Yes, because for $k = 2$, you go over 1 and up 2. For $k = \frac{1}{2}$, the unit rate is $\frac{1/2}{1}$ and you go over 1 but only up $\frac{1}{2}$.
• ***A line generated by a unit rate of 4 to 1 will be the same as the line generated by a slope triangle that goes over 4 and up 1.***	Answer: No, the rate for the slope triangle over 4 and up 1 would be $\frac{1/4}{1}$, which is not the unit rate $\frac{4}{1}$.

Additional Discussion (continued)

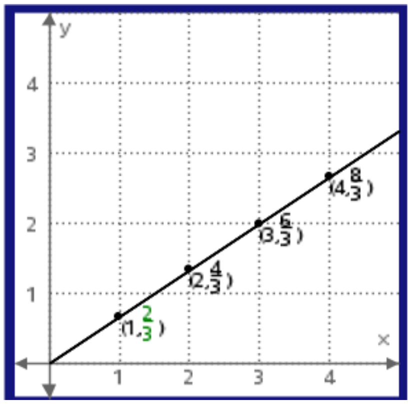
- ***Equivalent ratios have the same unit rate.***

Answer: Yes, because equivalent ratios are of the form cA:cB and the associated unit rate would be $\frac{cB / cA}{1}$, which is the same as $\frac{B / A}{1}$.

Sample Assessment Items

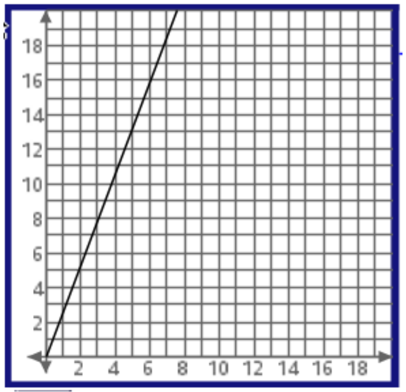
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

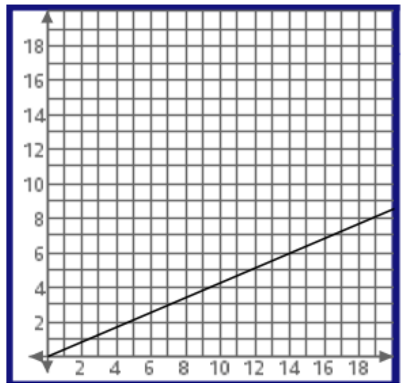
1. What is the equation of the line?

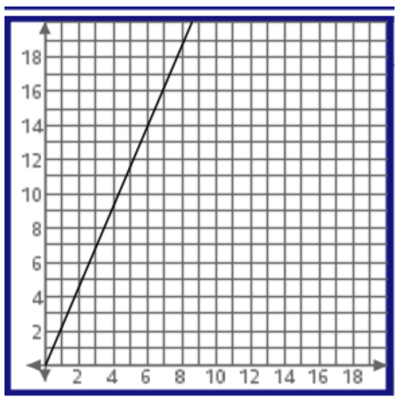


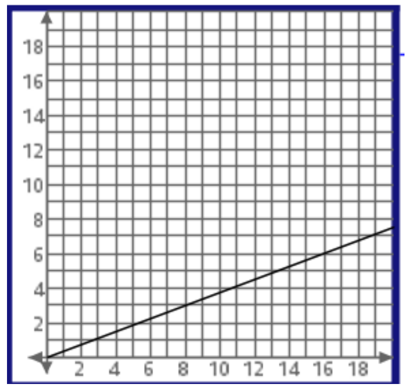
Answer: $y = \frac{2}{3}x$

2. Which of the following is the graph of the line $y = \frac{7}{3}x$?

a. 

b. 

c. 

d. 

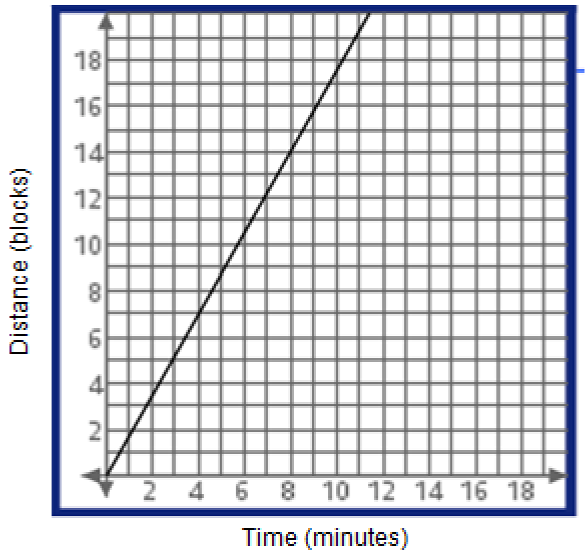
Answer: c

3. Order the equations below in terms of steepest slope to least steep.

a. $y = \frac{2}{3}x$ b. $y = 2x$ c. $y = \frac{1}{3}x$ d. $y = x$

Answer: b, d, a, c

4. The graph shows the distance Tonya walked in a given amount of time. How many blocks did she walk in 4 minutes?

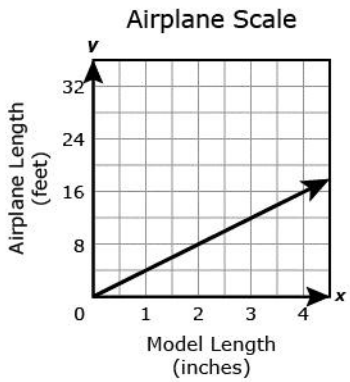


Answer: 7 blocks

5. Miranda made a model airplane using a scale in which $\frac{1}{4}$ inch represents 2 feet. Which graph shows this relationship?

adapted from Texas TEKS, 2013, Grade 7

A



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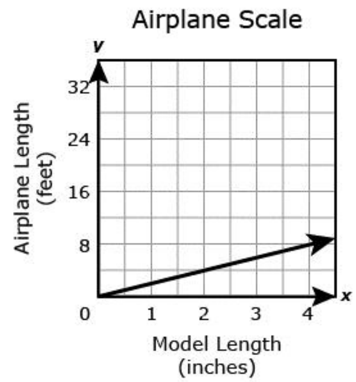
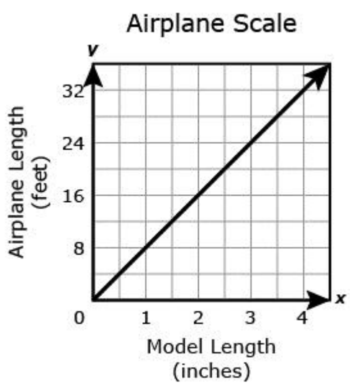


B



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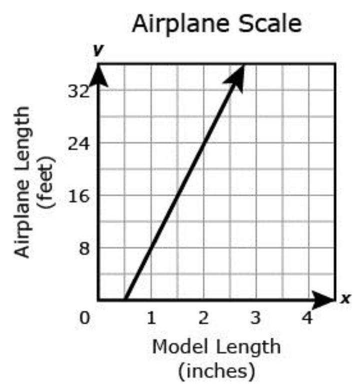

Answer: b

Building Concepts: Connecting Ratios to Equations

TEACHER NOTES

Student Activity Solutions

In these activities you will graph proportional relationships and use parallel grids to compare the tables, graphs, and equations. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. The ratio of the change in each *y*-value to a 1-unit change in the *x*-value for a graph of a proportional relationship is associated with a unit rate, the vertical increase in a "unit rate triangle" or "slope triangle" with horizontal side of length 1. The value is called the rate of change or the slope of the line.

   a. On page 1.3, describe a slope triangle and give the coordinates of the three points that determine the triangle.

   Possible answer: (0, 0), (1, 0,) and (1, $\frac{3}{2}$*)*

   b. Why do you think the triangle is called a "slope" triangle?

   Possible answer: The legs of the slope triangle indicate the horizontal and vertical change between two points on a line, which is the hypotenuse of the triangle. The slope triangle measures the steepness or the slope of the line.

2. General statements about relationships between quantities can often be expressed using symbols. An equation is given above the table on page 1.3.

   a. How is this equation related to the table and the graph?

   Answer: The equation $y = \frac{3/2}{1}x$ *says that every y-value in the table can be generated by the product of the x-value and the unit rate. The equation describes the points on the line formed by the ratios equivalent to 1:*$\frac{3}{2}$*.*

   b. The rate of change (slope) of the line representing a proportional relationship is called the constant of proportionality. What is the constant of proportionality for the line on page 1.3?

   Answer: $\frac{3/2}{1}$ *or* $\frac{3}{2}$*.*

3. Reset page 1.3. Move the point over 1 and up 3.

   a. What is the unit rate?

   Answer: $\frac{3}{1}$ *or 3.*

   b. Use the unit rate to predict the next two points on the graph and in the table. Check your prediction using the TNS lesson.

   Answer: (2, 6) and (3, 9).

Activity 2 [Page 1.5]

1. Work with a partner. One of you should use the graph on the left side of page 1.5 and the other should use the graph on the right side of page 1.5. Each student should generate at least 3 points on the line and in the table.

   a. How do the two tables compare?

   Answer: The tables have some of the same entries, but the table on the left side of page 1.5 has fractional entries as well as the same whole number entries.

   b. How do the graphs compare?

   Answer: The graphs seem to be the same line.

   c. How are the slope triangles alike and how are they different? What might explain any differences?

   Answer. The slope triangles on the left of page 1.5 have a base of 1 and a height of $\frac{3}{2}$; the slope triangles on the right of page 1.5 have a base of 2 and a height of 3. The ratios $\frac{3}{2}$:1 and 3:2 are equivalent, so although the slope triangles are different sizes, they produce the same set of points on the line. One triangle is kind of a scaled up version of the other.

   d. Compare the slope for the two equations.

   Answer: They are basically the same except on the left side of page 1.5, the multiplier of x is expressed as $\frac{3/2}{1}$, and on the right side of page 1.5 the multiplier is $\frac{3}{2}$. These are the same because any number divided by 1 is just that number.

2. Reset page 1.5, and then move the point in the graph on the left side of page 1.5 to (1, 2).

   a. With your partner, decide where to move the point in the graph on the right side of page 1.5 to a point not (1, 2) that will make the lines on each page contain the same points. Explain your reasoning.

   Answer: Any point of the form (a, 2a) because those points will be associated with ratios equivalent to 1:2.

   b. Use the TNS lesson to check your answer. How can you tell from the graphs that the lines contain the same points?

   Answer: You can tell from the graph that the lines contain the same points; some of the points in the tables will also be the same. You can also tell from the equations because $y = \frac{2}{1}x$ is the same as $y = 2x$ on the left of side page 1.5. For example, $y = \frac{4}{2}x$ on the right side is the same as $y = 2x$ on left side of page 1.5, so the equations describe the same relationship.

   c. Give two other points not on either of the tables that will lie on the lines.

   Possible answer: (8,16) and (10,20).

Activity 3 [Page 2.2]

1. Find the equation of each proportional relationships described below. Use the TNS lesson to help your thinking.

   a. It contains the point (2, $\frac{3}{4}$).

   Answer: $y = \frac{3}{8}x$

   b. The vertical change is 3 for every 2 units of horizontal change is 3.

   Answer: $y = \frac{3}{2}x$

   c. As the *x*-values in the table increase by 1, the *y*-values increase by 3.

   Answer: y = 3x

2. Given the ratio *a:b*

   a. What is the difference between the unit rate, the slope of a line, and the constant of proportionality associated with *a:b*?

   Answer: the ratio a:b is associated with the fraction $\frac{a/\ b}{1}$*, which is the unit rate; the slope of the line containing the set of ratios equivalent to a:b is* $\frac{b}{a}$*; and the constant of proportionality is the slope of the line,* $\frac{b}{a}$.

   b. How can you find the constant of proportionality from an equation, a table, and a graph?

   Answer: The constant of proportionality is the nonzero number c in the equation $y = cx$*, the change in the y-values in the table for every unit change in the x-values, and the slope of the graph of a line.*