Building Concepts: Fractions and Unit Squares

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson, essentially a dynamic geoboard, is intended to extend the concept of fraction to unit squares, where the unit fraction $\frac{1}{b}$ is a portion of the area of a unit square. In $\frac{a}{b}$, ***b*** indicates the number of regions—which have equal area—into which the unit square has been divided, and ***a*** represents the number of those regions that are shaded.

A unit on a number line can be related to a unit square with sides the same length as the unit on a corresponding number line.

Learning Goals

Students should understand and be able to explain each of the following:

1. A fraction $\frac{1}{b}$ is one part of a unit square that has been partitioned into ***b*** equal regions.
2. The fraction $\frac{a}{b}$ is ***a*** copies of $\frac{1}{b}$ on both a number line and in a unit square.
3. A fraction can be named in several ways.
4. A fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$.
5. How to order fractions with the same denominator.
6. The larger the denominator in a unit fraction, the more equally sized parts that can partition the unit square and the smaller the total area that the fraction represents.

Prerequisite Knowledge

Fractions and Unit Squares is the third lesson in a series of lessons that explore fractions. This lesson builds on the concepts explored tin the previous two lessons: *What is a Fraction?* and *Equivalent Fractions.* Students should be familiar with the following vocabulary: *unit fraction*, *equivalent fractions* and *improper fraction* covered in earlier lessons. Prior to working on this lesson students should understand:

- that the denominator in a fraction indicates the number of equal parts in each whole unit and the numerator indicates the number of those parts.
- that two fractions are considered equivalent if they are located at the same point on a number line.

Vocabulary

- **tiling:** using one or more shapes (the tiles) to cover a region without any gaps or overlaps.
- **unit square:** a square with side lengths equal to 1 unit.

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Fractions and Unit Squares_Student.pdf
- Fractions and Unit Squares_Student.doc
- Fractions and Unit Squares.tns
- Fractions and Unit Squares _Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

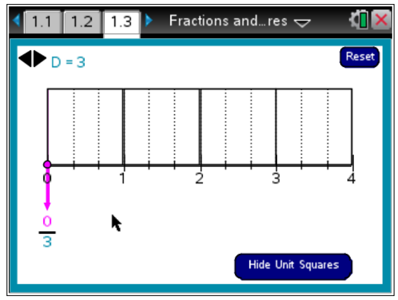
A unit on a number line can be related to a unit square with sides the same length as the unit on a corresponding number line. This TI-Nspire™ lesson, essentially a dynamic geoboard, is intended to extend the concept of fraction to unit squares, where the unit fraction $\frac{1}{b}$ is a portion of the area of a unit square. In $\frac{a}{b}$, ***b*** indicates the number of regions—which have equal area—into which the unit square has been divided, and ***a*** represents the number of those regions that are shaded. The lesson can be used to reinforce the concept of equivalent fractions by looking at different configurations of the congruent regions and also to illustrate the need to clearly specify the unit before a fraction can be interpreted or used with any meaning.

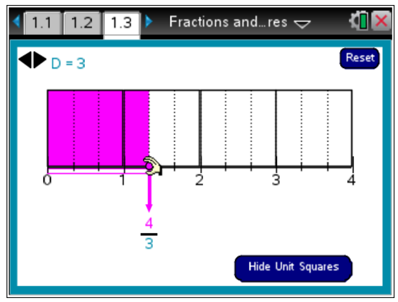
Students should have prior experience with the content in *What is a Fraction?* and *Equivalent Fractions*. Students should also recognize that the unit square has to be partitioned into regions of equal area. Some of the wording in the problems might need to be adjusted slightly depending on the background of the students. The language of tiling is one possible instance—a region is tiled by a shape if it is completely covered by the shape with no gaps or overlaps.

Part 1, Page 1.3

Focus: Students will develop the concept of unit squares.

In this activity a number line and unit squares are used to further investigate fractions. On page 1.3, the slider for D sets the denominator of a fraction. Drag the dot along the number line to set a numerator for the fraction with denominator D. Select **Show Unit Squares** to display the unit squares above the fraction line that correspond to the units on the number line; each side of the unit square is one unit in length. The unit squares are partitioned into D rectangles of equal area. In other words, the number of rectangles that partition the unit square is determined by the denominator of the fraction.





TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

The rectangles are shaded to represent the numerator. Just as $\frac{a}{b}$ represents ***a*** copies of $\frac{1}{b}$ on the number line, $\frac{a}{b}$ also represents ***a*** copies of rectangles of area $\frac{1}{b}$ on the corresponding unit square.

Teacher Tip: Give students time to repeat the activity before asking them a focused set of questions. This will help them internalize the concept of fractions and unit squares. Help students make the connection between the size of the rectangles in the unit squares and the denominator in the fraction. Guide students in a discussion about their exploration of unit squares.

Class Discussion

Have students…	Look for/Listen for…
***On page 1.3, set the fraction $\frac{1}{5}$. Then select* Show Unit Squares**.	
• ***What do you think is the area of a unit square? Of one of the rectangles? Why?***	Answer: The area of the unit square is 1 because the unit square has a base and height equal to 1, and the area of a rectangle is the product of the base and height. The area of one of the rectangles selected is $\frac{1}{5}$ of the area of the unit square because there are 5 rectangles, each the same size.
• ***How are the unit squares partitioned? How does this relate to the number line below the unit squares?***	Answer: The unit squares are partitioned into five rectangles of equal area, arranged horizontally. The partitioning matches the partition on the number line into fifths.
• ***What does $\frac{1}{5}$ represent on both the number line and the unit square?***	Answer: $\frac{1}{5}$ represents $\frac{1}{5}$ of the length from 0 to 1 on the number line beginning at 0 and it represents $\frac{1}{5}$ of the area of the unit square.
• ***Move the dot to display $\frac{4}{5}$. What does $\frac{4}{5}$ represent on both the number line and the unit square?***	Answer: The $\frac{4}{5}$ represents 4 copies of $\frac{1}{5}$ marked off on the number line, beginning at 0 and it represents 4 copies of a rectangle of area $\frac{1}{5}$ so it is $\frac{4}{5}$ of the area of the unit square.
• ***How do the five rectangles marked in each unit square compare to each other. Why is it important to think about this?***	Answer: The rectangles have the same area; they are each $\frac{1}{5}$ of a unit square; each one can be placed on top of any of the other rectangles and it will fit perfectly. If they had different areas, the unit square would not be partitioned equally and you could not talk about 4 copies of a rectangle as $\frac{4}{5}$ of the area.

Class Discussion (continued)

Have students…	Look for/Listen for…
Change D to 8 and select the fraction $\frac{11}{8}$.	
• ***What do you expect to see if you select*** **Hide Unit Squares*****?***	Answer: a number line with each unit partitioned into 8 pieces.
• ***Check your answer by selecting*** **Hide Unit Squares.** ***Will the rectangles making up the unit squares have the same area? How do you know?***	Answer: Yes the rectangles each have a base of $\mathbf{\frac{1}{8}}$ and a height of 1, so the area is the same for all of them.
Lydia said the fraction $\frac{7}{10}$ ***could be thought about as 7 copies of a region of a unit square that had each have an area of*** $\frac{1}{10}$***.***	
• ***Do you agree or disagree with Lydia? Explain your thinking.***	Possible answer: I agree because 7 copies of $\frac{1}{10}$ would be $\frac{7}{10}$, just like on the number line.
• ***Why is it important that the regions of the unit square all have the same shape and area*** $\frac{1}{10}$ ***?***	Possible answer: If they did not have the same area with the same shape, you could not count them as copies of the same thing.
Select the fraction $\frac{1}{2}$ ***in the unit squares.***	
• ***As the denominator of a unit fraction increases, what happens to the area represented by the fraction in the unit square? Use the file to verify your answer.***	Answer: The area gets smaller.
• ***Use your thinking in the question above and the file to decide which fraction is larger,*** $\frac{1}{10}$ ***or*** $\frac{1}{3}$***. Explain your answer.***	Answer: $\frac{1}{10}$ is smaller than $\frac{1}{3}$.

Class Discussion (continued)

Have students…

✓ ***Suppose you shaded unit squares to show an area representing $\frac{4}{3}$ and an area representing $\frac{5}{4}$. Explain by reasoning about the fractions and the unit squares which is larger and why. Use the file to check your answer.***

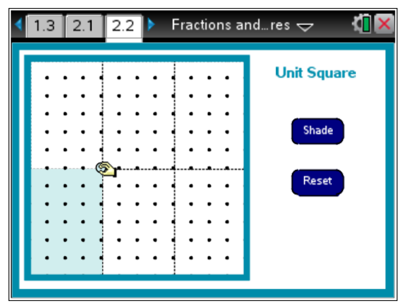
(Question #1 on the Student Activity sheet.)

Look for/Listen for…

Answer: $\frac{4}{3}$ is larger than $\frac{5}{4}$ because the fractions use up one whole unit square and $\frac{1}{3}$ and $\frac{1}{4}$ respectively, of a second square. Since $\frac{1}{3}$ into the second unit square takes more area than $\frac{1}{4}$, you can reason that $\frac{4}{3}$ is larger than $\frac{5}{4}$.

Part 2, Page 2.2

Focus: Students will investigate fractions as equal parts of a unit square.



On page 2.2, the large square framed in color is a unit square, which has been partitioned or tiled into 4 rectangles of equal area. Moving the dot in the middle of the unit square creates other ways to partition (tile) the unit square into rectangles with equal area. When the location of the dot produces such rectangles, the rectangles will be outlined with dotted lines.

Once the unit square has been divided into equal rectangles, select **Shade,** then select an individual rectangle to shade that rectangle. Shading can be removed by selecting that rectangle a second time. The **Reset** button is used to return the unit square to the starting configuration.

> **Teacher Tip:** Point out to students that the dot will need to be moved either vertically or horizontally or both vertically *and* horizontally to produce congruent rectangles.

As students work with or observe the manipulation of the unit square, help them to relate the total number of rectangles to the denominator of a fraction and the number of shaded rectangles to the numerator. Encourage students to discuss their findings.

Class Discussion

***On page 2.2, move the dot so the unit square is partitioned or tiled into 8 rectangles*.**

- ***What fraction of the area in the unit square does each rectangle cover?***

  Answer: $\frac{1}{8}$ of the area.

- ***Shade 3 of the rectangles. Explain what part of the unit square 3 of the rectangles would represent.***

  Answer: $\frac{3}{8}$ of the area of the unit square.

- ***Find three different ways to represent the fraction $\frac{5}{8}$ by shading some of the rectangles.***

  Make a sketch of your answers.
  Answer: Student sketches should show any 5 of the 8 rectangles shaded.

***Move the dot to tile the unit square into 6 rectangles*.**

- ***What fraction of the area in the unit square does each rectangle cover?***

  Answer: $\frac{1}{6}$

- ***Shade 5 of the rectangles. What fraction does this represent?***

  Answer: $\frac{5}{6}$ of the area of the unit square

- ***What would you expect to see in a unit square for the fraction $\frac{4}{6}$?***

  Answer: Any 4 of the 6 rectangles shaded.

Have students…	**Look for/Listen for…**
Tile the unit square into 16 rectangles. Create an argument that supports each claim:	
• $\frac{4}{16}$ ***is equivalent to*** $\frac{1}{4}$.	Possible answer: If you shade 4 rectangles in the top row of the unit square, you have shaded 4 of the 16 rectangles and you have shaded 1 of the 4 horizontal rows that make up the unit square, so you have shaded $\frac{1}{4}$ of the unit square.
• $\frac{6}{16}$ ***is equivalent to*** $\frac{3}{8}$.	Possible answer: If you pretend there is a line down the middle of the unit square, you can think of 4 pairs of 2 rectangles on either side

for 8 pairs of rectangles. If you shade the first 2 rectangles in the first three rows on the left side, you have shaded $\frac{6}{16}$ but also 3 of the eight pairs. So $\mathbf{\frac{6}{16} = \frac{3}{8}}$.

$\frac{5}{8}$ ***is equivalent to some fraction with a denominator of 16.***

Possible answer: Shading 5 pairs of the eight pairs described in 7b above will be $\frac{5}{8}$ and also $\frac{10}{16}$.

Class Discussion (continued)

Have students…

Look for/Listen for…

- ***Suppose you wanted to tile the unit square into 24 rectangles. How many rectangles would you try to have on each side? Explain your reasoning.***

Possible answer: (for a tiling with four rectangles vertically and six rectangles horizontally): $\frac{4}{6}$—sketch could show four of the 6 horizontal rows shaded; $\frac{8}{12}$— sketch could show 12 pairs and eight of them shaded; $\frac{2}{3}$—sketch could show 2 pairs of the 6 horizontal rows shaded and 1 pair of the 6 horizontal rows not shaded.

Explain your reasoning for each answer.

- ***What is the smallest fraction greater than 0 you can find using the unit square in the file?***

Answer: $\frac{1}{72}$ because 6 and 12 are the largest numbers you can use to tile the unit square, and that makes 72 small rectangles.

- ***What is the largest fraction less than one you can find using the unit square in the file?***

Answer: $\frac{1}{2}$ because all of the other unit fractions are smaller than $\frac{1}{2}$.

- ***Which is larger and why:*** $\frac{1}{2}$ ***or*** $\frac{1}{12}$***?***

Answer: $\frac{1}{12}$ is smaller than $\frac{1}{2}$. $\frac{1}{12}$ of the area of the unit square is less area than $\frac{1}{2}$ of the area.

Move the dot to partition the unit square into 12 rectangles.

- ***Shade $\frac{1}{2}$. What is $\frac{1}{6}$ of $\frac{1}{2}$? Explain your reasoning.***

  Answer: For $\frac{1}{2}$, six of the rectangles shaded. $\frac{1}{6}$ of these is one rectangle or $\frac{1}{12}$.

Class Discussion (continued)

- ***Shade $\frac{4}{12}$. Find a fraction on the unit square that is equivalent to $\frac{4}{12}$. Make a sketch to help explain why your claim is correct.***

  Possible answer: Shading the 4 rectangles in one of the three columns will be the same as $\frac{1}{3}$ of all the rectangles (for 3 across and 4 horizontal).

✓ ***Make a prediction about an equivalent fraction for $\frac{10}{24}$ whose denominator is 12. Use the activity to verify your prediction.***
(Question #2 on the Student Activity sheet.)

Answer: $\frac{5}{12}$.

For each pair of fractions identify the greater fraction and write a comparison statement using <, =, or >. Explain your reasoning using the number line.

- ***denominator of 4***

  Possible answer: (tiling is 3 across the bottom, 4 high): $\frac{3}{4}=\frac{9}{12}$; shade 3 of the 4 horizontal rows.
- ***denominator of 24***

  Possible answer: (tiling is 4 across the bottom and 6 high): $\frac{18}{24}=\frac{9}{12}$; shade 9 of the 12 on the top half and 9 of the 12 on the bottom half for 18 out of 24.
- **denominator of 36**

  Possible answer: $\frac{27}{36}=\frac{9}{12}$; find three sets of 12 (horizontally or vertically) and shade 9 of 12 of each set for 27 shaded out of 36.
- ***Describe in general terms how you can find an equivalent fraction to a given fraction for a specified denominator.***

  Possible answer: You have to use the factors of the denominators to figure out how to find the numerator. For 12 and 24, it was twice as much, so you had to double; for 12 and 36, it was three times as much so you had to triple. For 12 and 48, it would be 4 times as much so you would have to multiply by 4. For 12 and 4, 4 times 3 is12 so you had to look for a third to make it work.

Class Discussion (continued)

- ***Tile the unit square into 12 congruent rectangles. Use the file to help you order the following fractions from smallest to largest: Explain your reasoning.***

  $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{1}{2}$ $\frac{7}{12}$

  Answer: In order from smallest to largest - $\frac{1}{2}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$. If you think of them as $\frac{1}{12}$ s, then it would be $\frac{6}{12}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$.

Answer each:

- ***Tim and his sister ate 8 of 12 brownies in a pan. What fraction of the brownies did they eat?***

  Answer: $\frac{8}{12}$ or $\frac{2}{3}$ of the pan.

- ***A rectangular pizza was cut into 24 equal sized pieces. If Sari and her friend ate 6 of them, what fraction did they eat?***

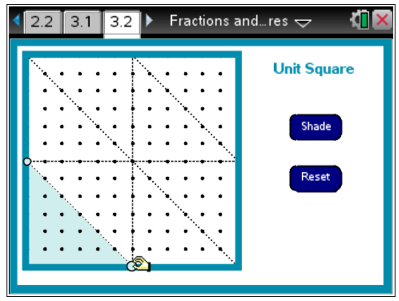
  Answer: $\frac{6}{24}$ or $\frac{1}{4}$.

- ***A cake was cut into 24 equal sized pieces. If 10 pieces were gone, what fraction was left?***

  Answer: $\frac{14}{24}$ or $\frac{7}{12}$ was left.

Part 3, Page 3.2

Focus: Students will further explore fractions using unit squares and congruent triangles.



Drag the dots along the vertical or horizontal segments to partition the unit square into different sets of triangles with equal area. Depending on the background of the students, it would be good to discuss why the triangles have the same area.

> **Teacher Tip:** You might take the opportunity to ask students how they can prove that the triangles have the same area.

Once the unit square has been partitioned into equal triangles, select **Shade** then select an individual triangle to shade it. Shading can be removed by selecting the triangle a second time. The **Reset** button is used to return the unit square to the starting configuration.

Lead students in a discussion about any patterns they see as they relate the number of triangles to the number of rectangles shaded in the unit square.

Class Discussion

- ***Shade in $\frac{1}{4}$. What is $\frac{1}{2}$ of $\frac{1}{4}$? Explain your reasoning.***

  Answer: $\frac{1}{8}$ because $\frac{1}{4}$ is 2 of the 8 triangles and $\frac{1}{2}$ of that would be 1 of the eight triangles or $\frac{1}{8}$ of the area.

Move the circles to tile the unit square with 16 triangles.

- ***Shade in $\frac{3}{16}$. What fraction is not shaded? Describe how you found your answer.***

  Answer: $\frac{3}{16}$ is not shaded. Subtracted 3 from 16.

- ***Shade $\frac{5}{16}$ on the bottom row of triangles and $\frac{3}{16}$ on the top row. What fraction of the unit square is shaded?***

  Answer: $\frac{8}{16}$ or $\frac{1}{2}$.

Class Discussion (continued)

✓ ***Shade 4 of the triangles. Give two names for the fraction of the unit square represented by the shaded area.***

(Question #3 on the Student Activity sheet.)

Answer: $\frac{4}{16}$ or $\frac{1}{4}$.

- ***Find another name for $\frac{6}{16}$. Use the file to justify your answer.***

  Answer: $\frac{3}{8}$.

Move the circles to tile the unit square with 24 triangles.

- ***Find two fractions equivalent to $\frac{8}{24}$.***

  Answer: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$

- ***Order the fractions from smallest to largest. Explain your thinking***.

  $\frac{11}{12}$, $\frac{9}{24}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{5}{8}$

  Possible answer: Thinking in terms of 24s, $\frac{8}{24}$, $\frac{9}{24}$, $\frac{12}{24}$, $\frac{15}{24}$, $\frac{20}{24}$, $\frac{22}{24}$; so the order would be $\frac{1}{3}$, $\frac{9}{24}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{5}{6}$, $\frac{11}{12}$.

Sample Assessment Items

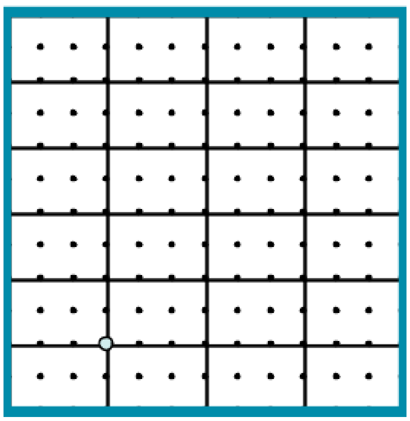
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Sean got 15 out of 24 questions correct on his math test. What fraction of the questions on the test did he answer correctly?

   a. $\frac{3}{4}$ b. $\frac{3}{5}$ c. $\frac{5}{6}$ d. $\frac{5}{8}$

   Answer: d

2. Using the unit square below, create a picture that shows $\frac{2}{3}$ is the same as $\frac{4}{6}$.

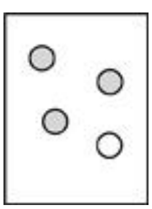


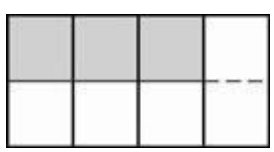
   Answer: Shade the bottom four rows.

3. Which picture shows that $\frac{3}{4}$ is the same as $\frac{6}{8}$?

a. 

b. 

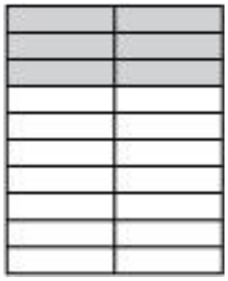
c. 

d. 

   Answer: a

NAEP 2009, Grade 4

4.



What fraction of the figure above is shaded?

NAEP 2007, Grade 8

a. $\frac{1}{4}$ b. $\frac{3}{10}$ c. $\frac{3}{7}$ d. $\frac{7}{10}$

Answer: b

5. Identify three fractions equivalent to $\frac{4}{12}$.

Answer: $\boldsymbol{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots}$

Student Activity solutions

Vocabulary

tiling:

using one or more shapes (the tiles) to cover a region without any gaps or overlaps.

unit square:

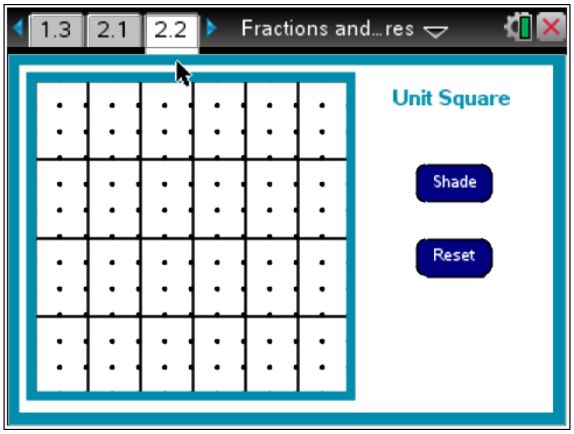
a square with side lengths equal to 1 unit

Students will use unit squares to name fractions and generate equivalent fractions.

1. Suppose you shaded unit squares to show an area representing $\frac{4}{3}$ and an area representing $\frac{5}{4}$. Explain by reasoning about the fractions and the unit squares which is larger and why.

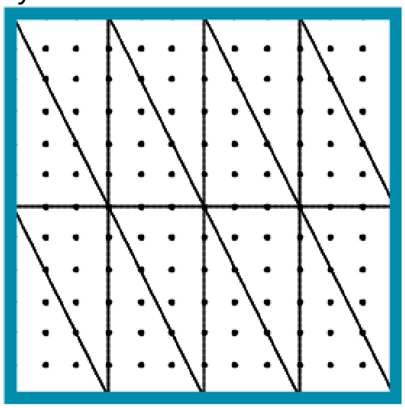
Answer: $\boldsymbol{\frac{4}{3}}$ ***is larger than*** $\boldsymbol{\frac{5}{4}}$ ***because fractions use up one whole unit square and*** $\boldsymbol{\frac{1}{3}}$ ***and*** $\boldsymbol{\frac{1}{4}}$ ***respectively, of a second square. Since*** $\boldsymbol{\frac{1}{3}}$ ***into the second unit square takes more area than*** $\boldsymbol{\frac{1}{4}}$ ***, you can reason that*** $\boldsymbol{\frac{4}{3}}$ ***is larger than*** $\boldsymbol{\frac{5}{4}}$***.***

2. Make a prediction about an equivalent fraction for $\frac{10}{24}$ whose denominator is 12. Use the activity to verify your prediction.



Answer: $\boldsymbol{\frac{5}{12}}$.

3. Shade 4 of the triangles. Give two names for the fraction of the unit square represented by the shaded area.



Answer: $\frac{4}{16}$ ***or*** $\frac{1}{4}$.

4. Can $\frac{4}{6}$ be equivalent to some fraction with a denominator of 12? Explain your reasoning.

Possible answer: A unit square that is divided into 6 pairs of 2 triangles shows both sixths (in rectangles) and twelfths (in triangles). If I shade the first 4 rectangles in the unit square then I show $\frac{4}{6}$ ***in rectangles, but also*** $\frac{8}{12}$ ***in triangles.***