Building Concepts: Dividing a Whole Number by a Fraction

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson unit squares are used to investigate division of whole numbers by a fraction. The focus is on how many groups of that fraction are in a given whole number. With a unit fraction, the result can be observed by partitioning the whole number. For example, 4 divided by $\frac{1}{3}$ is asking the question: how many $\frac{1}{3}$s are in 4. Each whole unit is partitioned into three $\frac{1}{3}$s; thus in 4 units, there are 3×4, or 12, $\frac{1}{3}$s (in general, ***c*** divided by $\frac{1}{b}$ will occur ***cb*** times).

When dividing a whole number ***c*** by a fraction $\frac{a}{b}$, the question asked is: how many groups of $\frac{a}{b}$s are contained in ***c***.

Learning Goals

Students should understand and be able to explain each of the following:

1. The division of a whole number by a unit fraction is the product of the whole number and the denominator of the fraction, i.e., $c \div \frac{1}{b} = c \times b$;
2. Dividing a whole number ***c*** by a fraction $\frac{a}{b}$ means you have to find how many groups or partial groups of $\frac{a}{b}$ are in ***c***;
3. Because multiplication and division are related, a division problem can be checked by considering its corresponding multiplication problem.

Prerequisite Knowledge

Students should have experience with the concepts in the lessons *Unit Squares and Fractions* and *Mixed Numbers.* Students should understand the relationship between multiplication and division and should have experience identifying a missing factor. The concepts in this lesson build upon the ideas presented in the previous lesson *Fraction Multiplication.* Prior to working on this lesson students should understand:

- how to solve to find a missing factor in a multiplication sentence.
- the concept of improper fractions and mixed numbers.

Vocabulary

- **perfect square:** the product of a number(an integer) multiplied by itself

Building Concepts: Dividing a Whole Number by a Fraction

TEACHER NOTES

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Dividing a Whole Number by a Fraction_Student.pdf
- Dividing a Whole Number by a Fraction_Student.doc
- Dividing a Whole Number by a Fraction.tns
- Dividing a Whole Number by a Fraction_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

In this TI-Nspire™ lesson, unit squares are used to investigate division of whole numbers by a fraction. The focus is on how many groups of that fraction are in a given whole number. With a unit fraction, the result can be observed by partitioning the whole number. For example, 4 divided by $\frac{1}{3}$ is asking the question: how many $\frac{1}{3}$s are in 4. Each whole unit is partitioned into three $\frac{1}{3}$s; thus in 4 units, there are 3×4, or 12, $\frac{1}{3}$s (in general, ***c*** divided by $\frac{1}{b}$ will occur ***cb*** times).

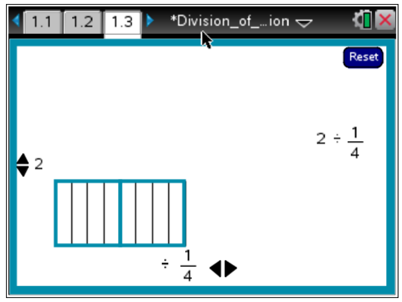
When dividing a whole number ***c*** by a fraction $\frac{a}{b}$, the question asked is: how many groups of $\frac{a}{b}$s are contained in ***c***. It is important to note that you are counting the number of groups of $\frac{a}{b}$. For example, to divide 4 by $\frac{2}{3}$, you consider how many groups of $\frac{2}{3}$ are in 4 units. In 1 unit, you have $1\frac{1}{2}$ groups of $\frac{2}{3}$s; in 2 units you have three groups of $\frac{2}{3}$s, so in 4 units you would have six groups of $\frac{2}{3}$. To divide 3 by $\frac{2}{3}$, you would have four groups of $\frac{2}{3}$s and half of another group. Thus your answer would be $4\frac{1}{2}$ or $\frac{9}{2}$. Because in some instances mixed numbers might be an obvious way to answer, students should be fluent in converting between improper fractions and mixed numbers.

Another approach to dividing a whole number by a fraction is to consider the relationship between multiplication and division, using the concept of a missing factor: $c \div \frac{a}{b} = d$ can also be expressed as $c = d \times \frac{a}{b}$. Students can rewrite division sentences as multiplication sentences and use the relationship between multiplication and division and the concept of a missing factor to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$. It might be helpful to review the lesson *Multiplying Fractions by Whole Numbers* when students use this approach.

Part 1, Page 1.3

Focus: Students will use unit squares to investigate dividing a whole number by a unit fraction.

Page 1.3 addresses the division of a whole number by a unit fraction using unit squares. One unit square is displayed, and the vertical arrows on the left generate up to 12 unit squares. The horizontal arrows at the bottom set a unit fraction and display the number of unit fractions in each unit square.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Demonstrate how division is represented on the unit square. Adjust the arrows to show $2 \div \frac{1}{4}$. Have students connect the whole number to the unit squares and the unit fraction to each partition within the unit square. Lead students to see that the quotient is the total number of partitions within the two unit squares.

Give students time to repeat the activity before asking them a focused set of questions. This will help them internalize the concept of dividing a whole number by a fraction. Encourage students to explain their reasoning for the answers.

Class Discussion

Have students…	Look for/Listen for…
One way to think about a divided by b is to count the number of possible b's in a. Use the unit square to explain the answer to the following:	
• $4 \div \frac{1}{4}$	Answer: The number of $\frac{1}{4}$s in 4 is 16
• $8 \div \frac{1}{4}$	Answers: The number of $\frac{1}{4}$s in 8 is 32

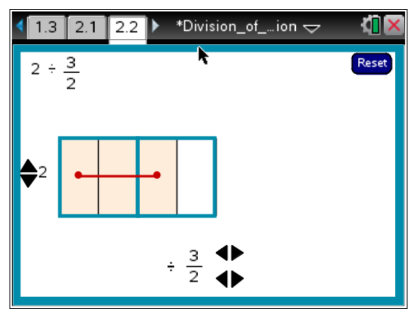
Class Discussion (continued)

- $8 \div \frac{1}{3}$ — Answer: The number of $\frac{1}{3}$s in 8 is 24
- $4 \div \frac{1}{3}$ — Answer: The number of $\frac{1}{3}$s in 4 is 12.
- ***5 divided by what unit fraction would give you an odd number less than 20?*** — Answer: $\frac{1}{3}$

Suppose your answer from dividing a whole number by a unit fraction is a perfect square. Give two examples where this would happen.

Answer: 3 divided by $\frac{1}{3}$; 2 divided by $\frac{1}{2}$; 4 divided by $\frac{1}{4}$, …

Part 2, Page 2.2



Focus: Students will continue to explore division of whole numbers by fractions.

Page 2.2 addresses the general case of the division of a whole number by a fraction. One unit square is displayed, and the vertical arrows to the left generate up to 12 unit squares. The horizontal arrows at the bottom set the denominator and numerator of a fraction and display the groups (or partial group) of the fraction in each unit. The groups are marked by line segments and alternate shading to make them easy to count.

Teacher Tip: Be sure students understand how the interaction with the unit square supports the mathematics. Asking them how the unit square is connected to their thinking about what division means, particularly with respect to whole numbers, can lead to a productive discussion that might surface their misunderstandings. Discussing the relationship between division and multiplication can reinforce a fundamental mathematical concept.

Have students…

Look for/Listen for…

- ***Use the unit square to divide 3 by $\frac{1}{4}$.***

Answer: 12

✓ ***Use the unit square to divide 3 by $\frac{3}{4}$. Explain the diagram this produces.*** (Question #1 on the Student Activity sheet.)

Answer: There are four groups of $\frac{3}{4}$ in 3. You can see the groups shaded and each group is marked with a line segment.

- ***How are your answers to the above questions related?***

Answer: There are 12 copies of $\frac{1}{4}$ in 3 but then you need to divide the 12 by 3 to see how many groups of $\frac{3}{4}$s there are.

- ***Divide 4 by $\frac{1}{5}$.***

Answer: 20

- ***Make a conjecture about the answer to dividing 4 by $\frac{2}{5}$. Dividing 4 by $\frac{4}{5}$.***

Answer 4 divided by $\frac{2}{5}$ will be 10; 4 divided by $\frac{4}{5}$ will be 5.

- ***How does the unit square on page 2.2 verify your conjectures?***

Answer: It shows 20 copies of $\frac{1}{5}$, which can be seen as 10 groups of or 5 groups of $\frac{4}{5}$s. If you are dividing by $\frac{2}{5}$, there are 10 groups of 2 partitions, each with a size of $\frac{2}{5}$.

Use the unit square to investigate the following:

- ***How many whole groupings of $\frac{3}{5}$ are in 4? What is left over?***

Answer: there are 20 copies of $\frac{1}{5}$ in 4; there are 6 groups of $\frac{3}{5}$, which uses 18 of the copies. There are 2 copies of $\frac{1}{5}$ left over.

- ***How many more $\frac{1}{5}$'s would you need to have another whole group of $\frac{3}{5}$?***

Answer: You would need one more $\frac{1}{5}$ to have a complete group of $\frac{3}{5}$s.

Class Discussion (continued)

✓ ***Explain why the answer to 4 divided by*** $\boldsymbol{\frac{3}{5}}$ ***is*** $\boldsymbol{6\frac{2}{3}}$.

(Question #2 on the Student Activity sheet.)

Answer: There are 20 copies of $\frac{1}{5}$ in 4. There are 6 whole groups of $\frac{3}{5}$s, which takes up 18 of the copies and you have $\frac{2}{3}$ of a group left, which are the remaining 2 copies from the twenty.

Explain why the answer to 6 divided by $\boldsymbol{\frac{3}{5}}$ ***is a whole number.***

Answer: 6 divided by $\frac{1}{5}$ is 30; i.e., there are 30 copies of $\frac{1}{5}$ in 6. There are 10 groups of $\frac{3}{5}$ in the 30 copies, so the answer is 10. The reason the answer is a whole number is that the number of copies of the unit fraction (30) was divisible by the number of $\frac{1}{5}$s (3) in the groups you were making.

Use the unit square to help answer the following:

- ***How many whole groups of*** $\boldsymbol{\frac{3}{4}}$ ***are in 4? What is left over?***

  Answer: There are 16 copies of $\frac{1}{4}$ in 4. There are 5 groups of $\frac{3}{4}$s and one $\frac{1}{4}$ remaining. You would need three $\frac{1}{4}$s in order to make another whole group.

- ***What is the role of the 3 in figuring out the answer to*** $\boldsymbol{4 \div \frac{3}{4}}$***?***

  Answer: The 3 tells you how to group the copies of $\frac{1}{4}$.

- ***What is*** $\boldsymbol{4 \div \frac{3}{4}}$***? Explain why the answer makes sense***

  Answer: $5\frac{1}{3}$. The answer makes sense because there are 5 whole groups of $\frac{1}{4}$ in the 16 copies of $\frac{1}{4}$ and you have one $\frac{1}{4}$ remaining. You need three $\frac{1}{4}$s in order to create another complete group.

- ***Is the answer to $4 \div \frac{4}{3}$ a whole number? How can you tell?***

Answer: There are 12 copies of $\frac{1}{3}$ in 4 and you are grouping by 4s. So there are three groups of four $\frac{1}{3}$s in 12. The answer is a whole number, 3.

Class Discussion (continued)

- ***Use the unit square to find three problems where a whole number divided by a fraction not equal to 1 has a whole number answer.***

Possible answers: $3 \div \frac{12}{8} = 2$; $3 \div \frac{6}{8} = 4$; $3 \div \frac{1}{2} = 6$; $3 \div \frac{3}{2} = 2$; $3 \div \frac{3}{7} = 7$; $4 \div \frac{2}{7} = 14$; or by a fraction equivalent to those given.

Answer each and explain your reasoning in each case.

- ***How many $\frac{1}{3}$-cup servings of raisins are in 3 cups?***

Answer: nine $\frac{1}{3}$-cup servings

✓ ***How many $\frac{2}{3}$-cup servings of cereal are in 3 cups?***
(Question #3 on the Student Activity sheet.)

Answer: $4\frac{1}{2}$ servings, each $\frac{2}{3}$ cup

- ***How many $\frac{1}{4}$-pound servings of meat are there in 4 pounds?***

Answer: 16 servings, each $\frac{1}{4}$ pound

The statement $6 \div 3 = 2$ can be rewritten as $6 = 3 \times 2$.

- ***Rewrite the statement $3 \div \frac{4}{5} = x$ as a multiplication problem.***

Answer: $3 = \frac{4}{5}(x)$

- ***Find the value of x. You may want to use the activity Multiplying Fractions by Whole Numbers to check your answer.***

Answer: $x = \frac{15}{4}$

- ***What is $3 \div \frac{3}{5}$? Explain how your work in the previous question can help you find the answer.***

Answer: 5 because when the divisor was $\frac{4}{5}$ the answer was $\frac{15}{4}$, so when it is $\frac{3}{5}$, the answer should be $\frac{15}{3}$ or 5.

- ***Tami announced that to check your answer for dividing a whole number by a***

Possible answer: Tami is correct because every division problem can be rewritten as a multiplication

fraction, you can find the product of the divisor and your answer. Sorn thinks you should use a rule that says invert and multiply. What would you say to Tami and Sorn?

problem. Sorn's rule is not very clear about what to invert and what it means to invert.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Consider the following problem: You used $2\frac{2}{3}$ cups of sugar to make 4 batches of pancake batter.

   How many cups would you use for 1 batch of batter?

   a. Is this problem a fraction divided by a whole number or a whole number divided by a fraction? Explain how you know. ***Answer: This is a fraction divided by a whole number because you have the number of cups for 4 batches and you want to figure out what it would be for 1 batch. You need to divide $2\frac{2}{3}$, the amount for 4 batches, into 4 parts.***

   b. What is the answer? ***Answer: $\frac{2}{3}$ cup per batch.***

2. Order the answers from each division problem from smallest to largest.

   a. $3 \div \frac{1}{6}$ b. $3 \div \frac{5}{6}$ c. $3 \div \frac{3}{4}$ d. $3 \div \frac{4}{3}$ e. $3 \div \frac{3}{6}$

   Answer: d, b, c, e, a

3. Identify each statement as true or false.

   a. $2 \div \frac{7}{8}$ is smaller than 3. ***Answer: True***

   b. If $5 \div \frac{2}{3} = a$, then $5 = \frac{2}{3} \times a$. ***Answer: True***

   c. If $a \div \frac{1}{10} = 60$, then $a = 6$. ***Answer: True***

   d. $4 \div \frac{5}{8} = 8 \div \frac{10}{16}$. ***Answer: False***

4. For the choices in problem 3 that are not true, change them to make a true statement. ***Possible answer:*** ***d.*** $4 \div \frac{5}{8} = 4 \div \frac{10}{16}$

Student Activity solutions

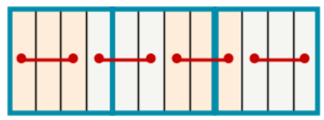
Vocabulary

perfect square:

the product of a number (an integer) multiplied by itself

In this activity, you will use a unit square and the relationship between multiplication and division to divide a fraction by a whole number.

1. What is the solution to 3 divided by $\frac{3}{4}$? Complete the diagram to show the division.

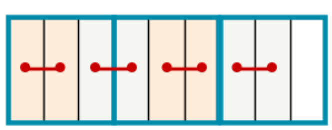


Answer: There are four groups of $\frac{3}{4}$ in 3. You can see the groups shaded and each group is marked with a line segment.

2. Explain why the answer to 4 divided by $\frac{3}{5}$ is $6\frac{2}{5}$.

Answer: There are 20 copies of $\frac{1}{5}$ in 4. There are 6 whole groups of $\frac{3}{5}$s, which takes up 18 of the copies and you have $\frac{2}{5}$ of a group left, which are the remaining 2 copies from the twenty

3. How many $\frac{2}{3}$-cup servings of cereal are in 3 cups? Complete the diagram to show your reasoning.



Answer: $4\frac{1}{3}$ serving, each $\frac{2}{3}$ cup

4. Hiro said that the answer to 8 divided by $\frac{2}{3}$ is a whole number. Is Hiro correct? Explain your answer.

Answer: Yes, Hiro is correct. 8 divided by $\frac{1}{3}$ is 24; so, there are 24 copies of $\frac{1}{3}$ in 8. There are 12 groups of $\frac{2}{3}$ in the 24 copies, so the answer is a whole number, 12. The reason the answer is a whole number is that the number of copies of the unit fraction (24) was divisible by the number of $\frac{1}{3}$ s (2) in the groups you were making.