Building Concepts: Adding Fractions with Unlike Denominators

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson uses unit squares to help students investigate addition and subtraction of fractions with unlike denominators. Students should recall that only fractions with a common denominator can be added. So, they can use their prior knowledge to add fractions with unlike denominators by finding a common denominator.

Fractions with unlike denominators need to be re-expressed in terms of equivalent fractions with a new common denominator.

Learning Goals

Students should understand and be able to explain each of the following:

1. When adding fractions, the fractions are marked off end to end on a number line. If the denominators are different, the total number of unit fractions cannot be counted because the unit fractions are of different sizes;
2. Fractions with uncommon denominators can be added by creating equivalent fractions that have the same denominator, then use the process for adding fractions with like denominators;
3. Many different common denominators can be found for any two fractions;
4. The sum of two fractions can be reduced if both the numerator and denominator of the sum have a common factor;
5. The sum (or difference) of two fractions with uncommon denominators in general, can be found by the formula $\left(\frac{a}{b}\right) \pm \left(\frac{c}{d}\right) = \left(\frac{ad}{bd}\right) \pm \left(\frac{bc}{bd}\right) = \frac{(ad \pm bc)}{bd}$

Prerequisite Knowledge

Adding Fractions with Unlike Denominators is the seventh lesson in a series of lessons that explore fractions and build on the concepts in previous lessons. Students should be familiar with the terms *unit fraction*, *equivalent fraction*, *common denominator, improper fraction, tiling,* and *unit square* covered in earlier lessons. Prior to working on this lesson students should understand:

- how to add and subtract fractions with like denominators.
- the concept of fractions on a number line.
- how to generate equivalent fractions.

Vocabulary

- **congruent**: any two shapes are congruent if, through a series of rigid motions, one can be superimposed on the other; the shapes will have equal area
- **common factors**: the same factors used to produce 2 or more numbers

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Adding Fractions with Unlike Denominators_Student.pdf
- Adding Fractions with Unlike Denominators_Student.doc
- Adding Fractions with Unlike Denominators.tns
- Adding Fractions with Unlike Denominators_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

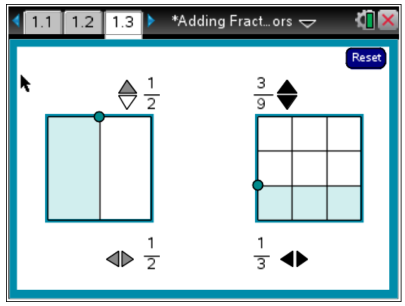
This TI-Nspire™ activity uses unit squares to help students investigate addition and subtraction of fractions with unlike denominators. Students should recall that only fractions with a common denominator can be added. So, they can use their prior knowledge to add fractions with unlike denominators by finding a common denominator. Note that the objective is to develop a simple, generalized method for finding the sum of two fractions. Finding a least common denominator is not necessary and, in fact, can lead to confusion for students, obscuring the goal of adding fractions. The files can also be used to investigate the properties for addition (commutative, associative, identity, inverse).

A fundamental concept is that a given unit square should be divided into *n* congruent pieces in order for $\frac{1}{n}$ to represent a unit fraction for the square. A second unit square can be divided into *n* congruent pieces that are not congruent to the pieces in the first square but $\frac{1}{n}$ still represents a unit fraction for that square. For example, two unit squares can each be divided into 12 congruent rectangles, but the rectangles in one square have dimensions $\frac{1}{6}\times\frac{1}{2}$ and in the other square the dimensions are $\frac{1}{3}\times\frac{1}{4}$. In both squares, a unit fraction will be $\frac{1}{12}$.

Part 1, Page 1.3

Focus: Students will use unit squares to add fractions with unlike denominators.

In this activity interactive unit squares are used to help students investigate adding fractions with unlike denominators. Page 1.3 displays two unit squares representing the fractions to be added. The arrows on the bottom set the denominator of each fraction. Moving the dots on the unit squares sets the numerators of the fractions. The arrows on the top of the page multiply the numerator and denominator of the fraction by a common factor from 2 to 12 and display the appropriate tiling of the unit square.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Students should be encouraged to make the connection between the original fraction and the equivalent fraction. Help them recall why it is possible to add the two fractions when the denominators are the same. Lead them in a discussion about the need for a common unit fraction in order to join one fraction end to end with the other on the number line and to be able to find the total number of unit fractions marked off.

If the denominators do not have a common factor, the rectangles in both unit squares are congruent. If the denominators have a common factor, however, the rectangles are not necessarily congruent, as displayed in the file. For example, to add $\frac{2}{3}+\frac{4}{9}$, with 27 as a common denominator, both unit squares are tiled into 27 congruent rectangles and the sum would be $\frac{30}{27}$. With 9 as the common denominator, both unit squares are still tiled into 9 congruent rectangles; and thus shading one rectangle in either unit square would be $\frac{1}{9}$ of the unit square. But $\frac{6}{9}$ produces a unit square tiled by rectangles of dimension $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{4}{9}$ is tiled by rectangles of dimension $1\times\frac{1}{9}$. The rectangles in the two unit squares have equal areas and each rectangle represents $\frac{1}{9}$ of the area, but the rectangles are not congruent. If this occurs in student answers, the difference should be discussed with students, helping them make connections to area and geometry.

Teacher Tip: Briefly review the concept of congruency. Use whole numbers and drawings to illustrate how two rectangles can have the same area, but not be congruent.

Class Discussion

Have students…	**Look for/Listen for…**
Use the arrows on the bottom to display the fraction $\frac{1}{11}$ in both of the unit squares.	
• ***Compare the areas representing $\frac{1}{11}$ in both of the unit squares. What do you notice?***	Possible answer: Even though one has horizontal rectangles and the other vertical, the areas are the same because the rectangles both represent $\frac{1}{11}$ of the area of the same-size unit square.
✓ ***Move the dots to show $\frac{5}{11}$ in the left unit square and $\frac{2}{11}$ in the right unit square. What is $\frac{5}{11}+\frac{2}{11}$? Explain how the display supports your answer.*** (Question #1 on the Student Activity sheet.)	Possible answer: The sum is $\frac{7}{11}$ because all of the rectangles are congruent, and each shaded rectangle represents $\frac{1}{11}$ of the area of the unit square.
Use the arrows and the dots to create the fractions $\frac{3}{4}$ and $\frac{2}{7}$.	
• ***Use the display to explain why you cannot add $\frac{3}{4}$ and $\frac{2}{7}$.***	Possible answer: The rectangles partitioning one of the squares are each $\frac{1}{4}$ of the unit square while the rectangles in the other are each $\frac{1}{7}$ of the unit square. If you combine them, you will not know what the unit fraction will be. They represent different things and so cannot be added.

Class Discussion (continued)

- ***Use the arrows at the top to change $\frac{3}{4}$ to its equivalent $\frac{6}{8}$ and $\frac{2}{7}$ to its equivalent $\frac{4}{14}$. Is the sum of $\frac{6}{8}$ and $\frac{4}{14}$ the same as $\frac{10}{22}$? Why or why not?***

Possible answer: They are not the same. You cannot add fractions unless they have a common denominator.

- ***Use the arrows on the top to create different fractions equivalent to $\frac{3}{4}$. Write your fractions in the table. Do the same for $\frac{2}{7}$, then compare the two sets of equivalent fractions.***

Answer:

3	6	9	12	15	18	21	24
4	8	12	16	20	24	28	32

2	4	6	8	10	12	14	16
7	14	21	28	35	42	49	56

- ***Are there equivalent fractions in the table that would allow you to add the two fractions? Explain.***

Possible answer: The fractions are both equivalent to fractions with denominators of 28. If the fractions have the same denominators you can add them.

- ***Using the information from the table and the file, find the sum of $\frac{3}{4}$ and $\frac{2}{7}$.***

  How does the file help you see the answer?

Answer: $\frac{3}{4}$ is equivalent to $\frac{21}{28}$ and $\frac{2}{7}$ is equivalent to $\frac{8}{28}$. The sum would be $\frac{29}{28}$. The file shows 21 small rectangles each $\frac{1}{28}$ of the unit square and 8 small rectangles also each $\frac{1}{28}$ of the other unit square. All together there are 29 small rectangles each $\frac{1}{28}$ of the unit square.

Class Discussion (continued)

For each of the following, make a conjecture about the common denominator, perhaps using a table as in problem 2. Then check your conjecture using the file to find the answer.

- $\frac{1}{2}+\frac{1}{3}$

  Answer: $\frac{5}{6}$.
- $\frac{5}{10}+\frac{2}{3}$

  Answer: $\frac{35}{30}$.
- $\frac{7}{8}+\frac{3}{4}$

  Answer: $\frac{13}{8}$ or $\frac{26}{16}$.
- $\frac{5}{6}+\frac{3}{4}$

  Answer: $\frac{38}{24}$ or an equivalent fraction.

✓ ***Is $\frac{3}{4}+\frac{2}{5}$ the same as $\frac{5}{9}$? Why or why not?*** (Question #2 on the Student Activity sheet.)

Answer: They are not the same. You cannot add denominators; you must find a common denominator in order to add fractions.

- ***Use the two unit squares in the file to decompose $\frac{13}{12}$ into the sum of two fractions with different denominators in at least two ways.***

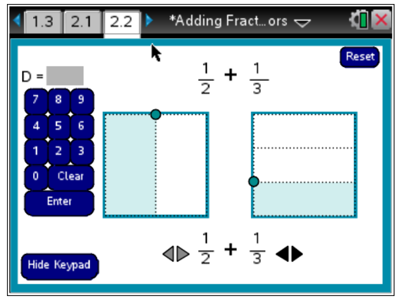
  Answer: $\frac{1}{12}+1$; $\frac{1}{6}+\frac{11}{12}$; $\frac{1}{4}+\frac{5}{6}$; $\frac{1}{3}+\frac{3}{4}$; $\frac{5}{12}+\frac{2}{3}$; $\frac{1}{2}+\frac{7}{12}$.
- ***Find two fractions with different denominators that add up to a whole number. What observation can you make?***

  Possible answer: $\frac{1}{3}+\frac{4}{6}$; $\frac{6}{18}+\frac{4}{6}$;…
- ***What fraction with a denominator different from 12 could you add to $\frac{5}{12}$ that would sum to $\frac{13}{12}$?***

  Answer: Any fraction equivalent to $\frac{2}{3}$ except $\frac{8}{12}$.

Part 2, Page 2.2

Focus: Students will find common denominators to add fractions.



On page 2.2, fractions are set in the same way as on page 1.3. The **Show Keypad** displays a keypad that can be used to enter a common denominator. If the denominator is not correct, the screen indicates that the number chosen is not a common denominator. Students should begin to see that a common denominator will always be the product of the two denominators. They might note that a smaller denominator sometimes works, but it is not necessary to make this an important concept in the discussion. The goal is to first recognize the need for a common denominator. To reset the page, select **Reset** in the upper right corner.

As students explore finding common denominators, encourage them to make predictions and explain their reasoning.

Class Discussion

Make a conjecture about the denominator that will allow you to add the two fractions. Check your answer using the file and find the sum of the fractions.

- $\frac{5}{9}+\frac{3}{10}$

  Answer: $\frac{77}{90}$.

- $\frac{3}{11}+\frac{1}{3}$

  Answer: $\frac{20}{33}$.

- $\frac{5}{8}+\frac{3}{4}$

  Answer: any fraction equivalent to $\frac{11}{8}$.

- $\frac{5}{12}+\frac{5}{8}$

  Answer: any fraction equivalent to $\frac{25}{24}$.

Class Discussion (continued)

Have students…	Look for/Listen for…
• ***Make a conjecture about which of the following denominators can be used to find $\frac{5}{9}+\frac{8}{12}$. Explain your thinking.*** 9, 12, 21, 36, 48, 72, 108	Possible answer: 36, 72, 108 because both 9 and 12 divide each of these numbers evenly; each is a multiple of both 9 and 12.
***Identify each as true or false. If the statement is false, explain why*.**	
• ***To add fractions with denominators 4 and 8, you can use a common denominator of 16.***	Answer: True.
• ***To add fractions with denominators of 3 and 4, you can use 7 as a common denominator.***	Answer: False because there are no equivalent fractions to $\frac{1}{3}$ and $\frac{1}{4}$ that have a denominator of 7.
• ***If you add the fractions $\frac{2}{5}$, $\frac{1}{2}$ and $\frac{3}{4}$, a common denominator could be 20.***	Answer: True.

Answer each of the following and explain your thinking.

- ***Sammy ran $\frac{3}{4}$ of a mile, stopped for water, and then ran another $\frac{5}{8}$ of a mile. How far did he run all together?***

  Answer: $\frac{11}{8}$ miles.

- ***The cook had $\frac{2}{3}$ of a pie. He served $\frac{5}{9}$ of that to customers. How much of the pie did he have left?***

  Answer: $\frac{1}{9}$.

Class Discussion (continued)

Have students…

Measuring cups come in the following sizes: 1 cup, $\frac{1}{2}$ cup, $\frac{1}{3}$ cup, and $\frac{1}{4}$ cup. How could you use a combination of the cups to measure out

Look for/Listen for…

- $\frac{1}{6}$ ***of a cup?***

Answer: Fill the $\frac{1}{2}$ cup and then pour as much of that into the $\frac{1}{3}$ cup as you can. What is left in the half cup will be $\frac{1}{6}$ because $\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$.

- $\frac{1}{12}$ ***of a cup?***

Answer: Fill the $\frac{1}{3}$ cup and then pour as much of that as you can into the $\frac{1}{4}$ cup. What is left in the $\frac{1}{3}$ cup will be $\frac{1}{12}$ because $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$.

- $\frac{5}{12}$ ***of a cup?***

Answer: Combine the $\frac{1}{3}$ and $\frac{1}{4}$ cup, which will give $\frac{7}{12}$ cup. Empty as much of that as you can into the half-cup. What is left will be $\frac{5}{12}$ cup because $\frac{7}{12}-\frac{6}{12}=\frac{1}{12}$.

✓ ***Which of the following will give you $\frac{4}{3}$ cups of sugar?***

(Question #3 on the Student Activity sheet.)

a. use a $\frac{1}{4}$ cup once, a $\frac{1}{3}$ cup once and a $\frac{1}{2}$ cup once

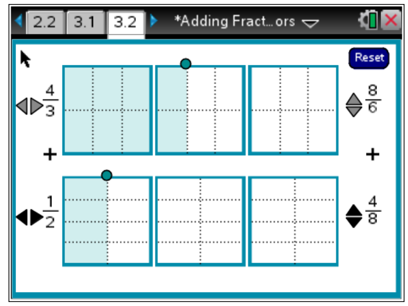
b. use a $\frac{1}{2}$ cup twice and a $\frac{1}{3}$ cup once

c. use a $\frac{1}{3}$ cup twice and a $\frac{1}{2}$ cup once

Answer: b.

Part 3, Page 3.2

Focus: Students will learn how to add improper fractions.



On page 3.2 the horizontal arrows at the left set the denominators of the fractions. Dragging the dots will set the numerator. The vertical arrows on the right generate equivalent fractions using factors from 2 to 12. Note that on this page, the rectangles in one unit square are not necessarily congruent to the rectangles in the second unit square. In the case of common denominators, each rectangle has been tiled by the same number of tiles, say *b*, so the shaded area represented by a rectangle in either of the unit squares is $\frac{1}{b}$, making addition possible.

In some cases, however, the rectangles do not look congruent because of orientation (For $\frac{3}{2}+\frac{4}{3}$ the denominator of 6 produces rectangles that are $\frac{1}{2}\times\frac{1}{3}$ and $\frac{1}{3}\times\frac{1}{2}$). In other cases, the rectangles in one unit square are not congruent to the rectangles in the other (i.e., a common denominator of 12 for $\frac{3}{2}+\frac{4}{3}$ tiles one unit square into 6 rows of 2 and the other into 4 rows of 3; in each case one rectangle is $\frac{1}{12}$ of the square but the rectangles themselves are not congruent). To reset the page, select **Reset** in the upper right corner.

> **Teacher Tip:** Remind students that an improper fraction is a fraction that is greater than 1. Lead them to see that they can quickly identify an improper fraction by looking at the numerator and denominator. If the numerator is greater than the denominator then the fraction is improper, or greater than 1.

Class Discussion

Have students…	Look for/Listen for…
Jaya said that to find the sum of $\frac{4}{3}$ and $\frac{8}{6}$, you could rewrite $\frac{8}{6}$ as $\frac{4}{3}$ and then you could add $\frac{4}{3}+\frac{4}{3}$ to get $\frac{8}{3}$. Jake said he thought you would rewrite $\frac{4}{3}$ as $\frac{8}{6}$ and add $\frac{8}{6}+\frac{8}{6}$ to get $\frac{16}{6}$. Who is correct and why?	Answer: They are both correct because $\frac{8}{3}$ is equivalent to $\frac{16}{6}$.

Class Discussion (continued)

- Which method could be used to add $\frac{7}{9}+\frac{3}{4}$?

  a. $\frac{4\times7+9\times3}{9+4}$ b. $\frac{7+3}{9+4}$ c. $\frac{4\times7+9\times3}{9\times4}$ d. $\frac{7\times3+9\times4}{9\times4}$

  Answer: c

Estimate each of the following to the nearest whole number. Check your answers using the file.

- $\frac{7}{8}+\frac{3}{4}$

  Answer: is about 2; $\frac{13}{8}$ or any equivalent fraction.

- $\frac{7}{3}+\frac{5}{6}$

  Answer: is about 3; $\frac{19}{6}$ or any equivalent fraction.

- $\frac{5}{7}+\frac{14}{11}$

  Answer: is about 2; $\frac{153}{77}$ or any equivalent fraction.

- $\frac{10}{9}+\frac{5}{8}$

  Answer: is about 2; $\frac{125}{72}$ or any equivalent fraction.

- ***Find a fraction with a denominator other than 3 you could add to $\frac{7}{3}$ to get a sum greater than 3 but less than 4.***

  Possible answers: $\frac{5}{6}$; $\frac{7}{8}$.

Class Discussion (continued)

Have students…	Look for/Listen for…
Without doing the calculations, decide which of the following is true or false. Explain your thinking.	
• $\frac{1}{3}+\frac{3}{8}<1$	Answer: True. $\frac{1}{3}$ plus something less than $\frac{1}{2}$ will not be more than 1.
• $\frac{5}{9}+\frac{6}{11}>1$	Answer: True. Both fractions are more than $\frac{1}{2}$ so the sum will be more than 1.
• $1-\frac{7}{12}>\frac{1}{2}$	Answer: False. $\frac{7}{12}$ is more than $\frac{1}{2}$ so subtracting from 1 will be less than $\frac{1}{2}$.
• $\frac{7}{9}+\frac{9}{8}>\frac{3}{2}$	Answer: True. $\frac{9}{8}$ is more than 1 and $\frac{7}{9}$ is more than $\frac{1}{2}$ so the sum will be more than $\frac{3}{2}$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Find each:

   a. $\frac{7}{8}-\frac{1}{2}$ ***Answer:*** $\boldsymbol{\frac{3}{8}}$.

   b. $\frac{11}{3}-\frac{7}{5}$ ***Answer:*** $\boldsymbol{\frac{34}{15}}$.

2. What number would you add to $\frac{5}{12}$ to get $\frac{3}{4}$? ***Answer:*** $\boldsymbol{\frac{4}{12}}$ ***or*** $\boldsymbol{\frac{1}{3}}$.

3. Given $\frac{1}{2}, \frac{7}{8}, \frac{11}{8}, \frac{2}{3}, \frac{1}{7}, \frac{3}{5}, \frac{11}{10}, \frac{2}{9}$ identify two different fractions whose sum is

   a. less than 1 ***Possible answer:*** $\boldsymbol{\frac{1}{2}+\frac{1}{7}}$.

   b. between 1 and 2 ***Possible answer:*** $\boldsymbol{\frac{11}{8}+\frac{1}{7}}$.

   c. greater than 2 ***Possible answer:*** $\boldsymbol{\frac{11}{10}+\frac{11}{8}}$.

4. If one part of a recipe calls for $\frac{2}{3}$ cup of milk and another part of the same recipe calls for $\frac{3}{4}$ cup of milk, how much milk do you need all together for the recipe? ***Answer:*** $\boldsymbol{\frac{17}{12}}$ ***cups***.

5. A recipe calls for $\frac{3}{4}$ cup of onions and you have only $\frac{3}{8}$ cup. How many more cups of onions do you need? ***Answer:*** $\boldsymbol{\frac{3}{8}}$ ***cup***.

6. Fill in the blank with the value that will make the statement true.

   a. $\frac{\square}{4}+\frac{2}{3}=\frac{11}{12}$ ***Answer: 1***.

   b. $\frac{7}{\square}-\frac{1}{2}=\frac{4}{6}$ ***Answer: 6***.

   c. $\frac{5}{8}-\frac{\square}{9}=\frac{5}{8}$ ***Answer: 0***.

7. Which shows a correct method for finding $\frac{1}{3}-\frac{1}{4}$?

   a. $\frac{1-1}{4-3}$ b. $\frac{1}{4-3}$ c. $\frac{3-4}{3\times 4}$ d. $\frac{4-3}{3\times 4}$

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Answer: d.

Student Activity solutions

Vocabulary

congruent:

any two shapes are congruent if, through a series of rigid motions, one can be superimposed on the other; the shapes will have equal area

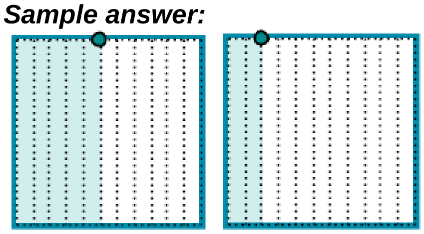
common factors:

the same factors used to produce 2 or more numbers

In this activity, you will create equivalent fractions to add fractions with unlike denominators.

1. What is $\frac{5}{11}+\frac{2}{11}$? Shade the unit squares to show the addition. Explain how the unit squares supports your answer.

Sample answer:



The sum is $\frac{7}{11}$ because all of the rectangles are congruent, and each shaded rectangle represents $\frac{1}{11}$ of the area of the unit square.

2. Is $\frac{3}{4}+\frac{2}{5}$ the same as $\frac{5}{9}$? Why or why not?

Answer: They are not the same. You cannot add denominators; you must find a common denominator in order to add fractions.

3. Which of the following will give you $\frac{4}{3}$ cups of sugar?

a. use a $\frac{1}{4}$ cup once, a $\frac{1}{3}$ cup once and a $\frac{1}{2}$ cup once

b. use a $\frac{1}{2}$ cup twice and a $\frac{1}{3}$ cup once

c. use a $\frac{1}{3}$ cup twice and a $\frac{1}{2}$ cup once

Answer: b.

4. Adam said that to find the sum of $\frac{6}{4}$ and $\frac{2}{3}$, you could rewrite $\frac{6}{4}$ as $\frac{6}{12}$ and $\frac{2}{3}$ as $\frac{2}{12}$ then you could add the two fractions together to get $\frac{8}{12}$. Is Adam correct? Explain why or why not.

Answer: Adam is incorrect. The correct way to rewrite $\frac{6}{4}$ is $\frac{18}{12}$, and the correct way to rewrite $\frac{2}{3}$ is $\frac{8}{12}$. The sum of $\frac{18}{12}$ and $\frac{8}{12}$ is $\frac{26}{12}$ or $2\frac{2}{12}$.