Building Concepts: Expressions & Equations Overview

Building Concepts: Expressions & Equations consists of a sequential, coordinated series of interactive, dynamic TI-Nspire™ activities designed to support a developmental trajectory of concepts related to expressions and equations. Beginning with expressions, students develop an understanding of variables and the concept of an expression. Students explore equations as true or false statements, which leads to the idea of solutions. Equations in more than one variable then lead to the development of relationships with a constraint. Finally students explore systems of simultaneous equations.

The activities are created for those who are teaching or learning about expressions and equations or reviewing the concepts, parents, or anyone interested in learning or helping others learn about expressions and equations. The activities are designed to develop students' expertise in the processes and proficiencies that serve as the foundation for the Standards for Mathematical Practice described in the Common Core State Standards.

Compatible TI Technologies

The TI-Nspire activities (TNS files) are compatible with the following technologies:

 TI-Nspire™ CX Handhelds,  TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software

The activities can be adapted for different classroom environments. One teacher might use a whiteboard to demonstrate the activity to a large group of students, while another might have students working individually or in small groups using a TI-Nspire handheld. The delivery method used depends on the technology available in the classroom. The estimated activity time for students working at grade level is 50–90 minutes, depending on the delivery method of the activity.

Each TI-Nspire activity (TNS) is accompanied by Teacher Notes and a Student Activity sheet to help facilitate the use of the TI-Nspire activity in the classroom.

Teacher Notes

The Teacher Notes are intended to provide a starting point for how these activities can be used with students. It is important to note that the TI-Nspire activities themselves are typically robust enough to be adapted for learning goals not mentioned in the Notes.

Teacher Notes are designed to serve as a guide for implementation in classrooms and include:

- descriptions of the mathematics that underlies each lesson
- a description of the activity and how to use it
- mathematical goals for student learning
- discussion questions for teacher-led classroom exploration or individual student investigation
- assessment questions that vary in difficulty. These questions might be found on the National Assessment of Educational Progress mathematic assessments or a high stakes state test.

Instructors may choose to use all of the suggested questions or select a few from those presented.

Student Activity

The Student Activity sheet is an optional resource that may be used as a tool to keep students engaged and on track throughout the lesson. This resource is particularly valuable in a large-group instruction scenario.

Although teachers can successfully work through the lessons without using the Student Activity sheet, the resource can prove to be valuable. The statistics and probability concepts align with the math standards for grades 6–8 and the activity sheet has been created to match this range. The Student Activity is provided as an editable word document and therefore may be modified to use with middle-school students or higher, increasing its value.

Lesson Series

The series consists of the following 18 lessons. Each lesson should take 50–90 minutes to complete with students, though teachers may choose to extend lessons, as needed.

1. *What is an Exponent?*

   In Grade 4 students gain familiarity with factors and multiples. In Grade 5 they used whole number exponents as a compact way to express powers of 10. In Grade 6 students expand these notions to use exponents as a compact way to write expressions involving whole number multiplication. This lesson focuses on how the definition of an exponent, the number of times a base is used as a factor ("when in doubt, write it out"), can be used in a variety of situations to make sense of the computation: $a^b, (a \cdot c)^b, (a^b)^c$. The lesson also explores the "distribution" of exponents over the operations of addition, subtraction and division.

2. *What is a Variable?*

   A mathematical *expression* is a phrase about a mathematical or real world situation. A mathematical expression can have one operation or a series of operations or be just a single number. Letters standing for numbers in an expression are called *variables*. In this lesson, students investigate mathematical expressions containing a variable and expressions that are constant. They evaluate expressions, write expressions that correspond to given situations, and use expressions and formulas to solve problems

3. *Building Expressions*

   A goal of this lesson is for students to see expressions as objects in their own right. They examine different expressions to determine which are equivalent. Their early use of properties to perform calculations with numbers, for example seeing 5 (47) as 5(40+7)=200+35 = 235, provides a basis for the transition to thinking about 4(*z*+10) as 4*z*+40 and to see 4*z*+40 as 4(*z*+10). They use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to support their reasoning.

4. *What is an Equation?*

   As described in Activity 3, *Building Expressions*, an expression is a phrase about a mathematical or real-world situation. An *equation* is a statement that two expressions are equal, such as $10 + 5n = 20$, or $5 + x = 4 + x$. A *solution* to an equation, if one exists, is a value that makes the equation true when substituted for the variable. In this activity students distinguish between expressions and equations. They investigate solutions to equations of the form ax+b = c, but the focus is on what an equation is and what it means to have a solution to an equation, not on finding a solution.).

5. *Equations and Operations*

   Solving an equation is a process of reasoning to find the numbers that make the equation true, which can include checking if a given number is a solution. If an equation has a solution, solving the equation is a search for that solution using a variety of reasoning strategies. The focus in this lesson is on developing an understanding of solution preserving moves that can be performed on an equation, transforming the equation into a new equation that will have the same solution as the original equation. (These solution preserving moves could be described as the addition and multiplication properties of equality.) This lesson investigates the relationship between the solution to an original equation and the solution to a new equation formed by adding, subtracting, multiplying or dividing one or both sides of the original equation by a numerical value.

6. *Using Structure to Solve Equations*

   This lesson extends students' strategies for solving equations by considering the structure of the relationships expressed by an equation and exploiting that structure to find a solution. Students "cover up" missing factors or missing addends and reason from their understanding of arithmetic to find the values of the missing quantities. Essentially the process reduces an equation to reasoning about a series of subexpressions that lead to the solution for the original equation

7. *Visualizing Equations Using Mobiles*

   Earlier lessons have given students tools to reason about and solve linear equations of the form $ax+b=c$. This activity extends their thinking to situations with a variable on both sides of the equation. The visual model of mobiles lends itself to thinking of equations as statements of balance and of solution preserving moves as related to maintaining balance.

8. *Linear Inequalities in One Variable*

   The goals of this lesson are to connect the algebraic representation of a linear inequality in one variable to a graphical representation of the solution to the inequality and to understand how techniques developed for solving equations can and cannot be used to find solutions to inequalities. Students also reason about compound inequalities and simple absolute values involving variables to develop notational fluency and understanding of the definitions these situations involve.

9. *Visualizing Linear Expressions*

In this lesson, students identify equivalent linear expressions by connecting them to different pictorial representations of the same entity. They consider the role properties, particularly the distributive property of multiplication over addition, play in creating equivalent expressions and identify and rewrite expressions as sums, as products, or as a combination of sums and products. They investigate how context can shed light on a problem and how the quantities in it are related.

10. *Building Expressions in Two Variables*

This lesson moves from *Building Expressions 1* introducing expressions that require understanding of the rules for operating with rational numbers (both positive and negative) and may involve two variables with rational coefficients. As students gain experience with multiple ways of writing an expression, they recognize that writing an expression in different ways can reveal a different way of seeing a problem and look for patterns in tables as a way to reason about expressions and their relationships. The activity can be extended to more complicated expressions of the second and third degree, where the goal is to prepare students for expanding and factoring polynomials

11. *Extending Exponents*

Students extend their knowledge of exponents to include integer exponents and use these for transforming expressions. They develop an understanding of how exponents behave by working with estimates of very large and very small quantities, typically written in the form of a single digit times a whole-number power of 10 and express how many times as much one is than the other as well as performing calculations with very large numbers using scientific notation. While in general, rational exponents are not considered in the activity, students learn to work with square and cube roots.

12. *Solving Equations*

Students have built a repertoire for reasoning about and solving equations that includes using the structure to find missing factors and addends and solution preserving moves. This activity takes a managerial approach where students inspect an equation and select an approach, and identify a series of steps which are carried out by the file. Students consider the efficiency and elegance of multiple approaches for finding the solution for a given equation with a goal of becoming fluent in solving linear equations in one variable that can be reduced to the form $ax+b=cx+d$.

13. *Equations of the form ax+by=c*

Not all equations in two variables are written explicitly in the form $y=mx + b$, and in many contexts, doing so distorts the meaning. The most general form of a line is ax+by=c because it includes vertical lines. In this activity students use coordinate grids to explore the “trade off” between the values of the two variables in order to maintain the constraint imposed by the constant c. They investigate the meaning of a solution in the context of a linear equation in two variables.

14. *Moving from Proportional Relationships to Linear Equations*

In studying ratios, students developed an understanding of proportional relationships. The constant of proportionality, *k*, arises as the multiplicative factor in an equation of the form $y=kx$ and can be interpreted as the slope of the line, and as the unit rate of change (the amount of change in the dependent variable resulting from 1 unit increase in the independent variable). In linear equations, the change in the dependent variable *y* remains proportional to the change in the dependent variable *x*. This activity builds on these ideas to develop the concepts of lines through the origin and of linear equations in general, focusing on the general form, $y=y_1+m(x-x_1)$, which can be transformed into the familiar $y=mx + b$.

15. *Visualizing Systems of Linear Equations*

This activity returns to the mobiles from *Visualizing Linear Equations Using Mobiles* to provide a visual problem solving experience related to solving systems of linear equations. Students revisit solution preserving moves and look for relationships among variables that will enable them to find solutions to two, three or four equations involving the same variables.

16. *What is a Solution to a System of Equations?*

In this activity the solution to a system of equations is presented as the point of intersection of the graphical representations of the equations. Students explore the systems with no solution, with one point as a solution and those that represent the same line whose solution is the set of all points on that line. They connect the graph of an inequality in two variables to its solution set, the graphical representation of the ordered pairs in the solution set in the coordinate plane. The graphical thinking related to the solution of compound inequalities in one variable is extended to finding the solution to a system of linear inequalities.

17. *Solving Systems of Equations Algebraically*

This activity focuses on solving systems of equations or inequalities using algebraic techniques guided by the strategy of reducing something unknown to something unknown. In this case finding ways to reduce two equations/inequalities in two unknowns to one equation/inequality in one unknown will result in a familiar situation, solving a linear equation in one variable. Students will decide whether the elimination method or substitution would be a more efficient approach and be able to justify their reasoning.

18. *Visualizing Quadratic Expressions*

In this activity, students use geometric representations to explore equivalent expressions that can be expressed in the form $x^2 + bx + c$ where *b* and *c* are positive integers. The emphasis is on using an area model to represent the product of two binomials, continuing to build on the interpretation of the distributive property of multiplication over addition as “each times every”: given the product of two binomials, each term in the sum of the first binomial multiplies every term in the sum of the second binomial.

Grade Level Alignment

It is recommended that the lessons be used in sequential order at any grade level. Each lesson builds upon a concept that has been presented in a previous lesson.

The following table shows how the lessons align with typical grade-level expectations.

	Grade 6		Grade 7		Grade 8	
	CCSS	**TEKS**	**CCSS**	**TEKS**	**CCSS**	**TEKS**
Lesson 1 *What is an Exponent?*	6.EE.A.2c			7.2.E		
Lesson 2 *What is a Variable?*	6.EE.A.2 6.EE.A.3	6.5.A		7.14.A		
Lesson 3 *Building Expressions*	6.EE.A.2 6.EE.A.3 6.EE.A.4	6.5.A 6.13.B				
Lesson 4 *What is an Equation?*	6.EE.A.4 6.EE.B.5 6.EE.B.6			7.5.B		
Lesson 5 *Equations and Operations*	6.EE.B.6 6.EE.B.7			7.5.A		
Lesson 6 *Using Structure to Solve Equations*	6.EE.B.5 6.EE.B.6 6.EE.B.7	6.10.A	7.EE.B.3 7.EE.B.4	7.11.A		
Lesson 7 *Visualizing Equations Using Mobiles*	6.EE.B.5	6.10.B 6.6.C 6.7.C		7.11.B		
Lesson 8 *Linear Inequalities in One Variable*	6.EE.B.5	6.10 .A	7.EE.B.4b	7.11.A		

	Grade 6		Grade 7		Grade 8	
	CCSS	TEKS	CCSS	TEKS	CCSS	TEKS
Lesson 9 *Visualizing Linear Expressions*	6.EE.A.2	6.10 .A	7.EE.A.2	7.11.A		
Lesson 10 *Building Expressions in Two Variables*		6.7.D	7.EE.A.2			
Lesson 11 *Extending Exponents*					8.EE.A.1	8.2.C
Lesson 12 *Solving Equations*		6.10.A		7.11.A	8.EE.C.7	8.8.C
Lesson 13 *Equations of the form ax+by=c*					8.EE.C.8	8.5.I
Lesson 14 *Moving from Proportional Relationships to Linear Equations*					8.EE.B.6	8.5.B
Lesson 15 *Visualizing Systems of Linear Equations*					8.EE.C.8	8.9
Lesson 16 *What is a Solution to a System of Equations?*					8.EE.C.8	8.9
Lesson 17 *Solving Systems of Equations Algebraically*					8.EE.C.8	8.9
Lesson 18* *Visualizing Quadratic Expressions*		6.7.C	7.EE.B.4			

** Lesson 18 also addresses the following Algebra 1 standards: A-SSE.A.2 (CCSS), A1.6 (TEKS)*

References

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

The Common Core Standards Writing Team. (2013, September 13). Progressions for Common Core State Standards in Mathematics 6-8 Statistics and Probability Retrieved from http://commoncoretools.me/wp-content/uploads/2011/12/ccss_progression_sp_68_2011_12_26_bis.pdf

Texas Essential Knowledge and Skills for Mathematics Subchapter A. Elementary, Chapter 111. (2012). Texas Education Agency